A COMPARISON AND EVALUATION OF RISK AND UNCERTAINTY TECHNIQUES FOR CAPITAL PROJECTS IN THE PUBLIC SECTOR.

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ABSTRACT

The basic problem facing decision makers is the allocation of scarce resources among competing users. Such decisions made are usually in the face of an essentially unknown future environment. This thesis is concerned with making good decisions under uncertainty in relation to public investment proposals in New Zealand agriculture.

A case is put for the evaluation of risk and uncertainty in agricultural projects, but on a review of evaluation techniques in use it is found that many deficiencies exist. In a search for a practical solution to the risk analysis problem it is shown that decision theory shows very little promise. Rather the solution is to make the risk and uncertainty explicit by presenting the present value in terms of its probability distribution. Two methods are discussed, they are Analytical Techniques and Monte Carlo Methods.

The analytical technique is developed through the use of probability calculus. Techniques evolve from being able to calculate the variance of simple summed random variables to handling complex combinations of products of random variables using Taylor's Approximation. Incorporated in the analysis are discussions on subjective probability distributions, forecasting techniques and correlation analysis. It is shown that the shape of the probability distribution is not nearly as important as the way in which

the variables relate to each other both within and between periods. Earlier criticisms of traditional techniques of handling risk and uncertainty are overcome. In the analytical technique judgment is applied to the underlying assumptions in the project rather than to the results of the analysis. The variability of the project is measured by a single overall indicator (the variance), not by a number of criteria. The technique allows for interaction between the variables which make up the project. A quantitative assessment of risk is made rather than qualitative statements and lastly the basic framework is laid for consistent analysis project to project and analyst to analyst.

Monte Carlo Simulation offers two major advantages over analytical methods. These are firstly, all the characteristics of the probability distribution of the variables can be simulated and secondly the dynamic aspects of project development can be incorporated into the analysis. There are, however, certain drawbacks to implementation. The major disadvantage is the time required to build a simulation model. The task of developing a general package was found to be beyond the scope of this thesis. The main problem encountered was in defining and incorporating the dependency relationships between variables.

A rural water supply scheme is analysed under several risk procedures to test the usefulness and practicability of each method. It is concluded that the analytical technique incorporating Taylor's approximation shows the most promise for implementation. However, before this can be done several factors require further investigation, the most critical factor being the specification of dependency relationships between stochastic variables.

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CHAPTER I

INTRODUCTION

1.1 PROBLEM FORMULATION

The basic problem facing decision makers is the allocation of scarce resources among competing users. Such decisions made are usually in the face of an essentially unknown future environment. This thesis is concerned with making good decisions under uncertainty in relation to public investment proposals in New Zealand agriculture.

1.2 BACKGROUND

During the late 1950's and early 1960's the Farm Management and Economics Section, of the New Zealand Department of Agriculture, was increasingly called upon to carry out land utilization and farm management surveys. The purpose of these was to assess the potential production increases arising from developments in farm water supply, irrigation, land reclamation and farm improvement schemes (see Table I).

The results of these surveys formed the basis of reports to the Ministry of Works and local authorities on which assessments could be made of the likely effect on farm production of expenditure on the schemes.

TABLE I. Investigations Carried out by the Department of Agriculture into Proposed Water Resource Projects[1]

Year ended 31st March	Irrigation	Water Supply	Conservation Drainage & River Control	Total
1952	1			1
1953				-
1954	1			1
1955		1		1
1956		1		1
1957	2	3		5
1958	2	1		3
1959		1		1
1960	7	2		9
1961	1			1
1962	1			1
1963	2	2		4
1964	1	2		3
1965	1	3		4
1966	1	7		8
1967	6	2	2	10
1968	6	6	4	16
1969	4	5	1	10
1970	4	8	4	16
1971	2	8	6	16
1972	2	10	10	22
1973	2	10	7	19

Source: Resource Section, Economics Division, M.A.F. Palmerston North

[1] The off-farm capital costs of these projects range from $2000 to over $15,000,000 with an average of $770,000.

In the early reports the accounting rate of return was used as the decision criterion. Discounting and the use of present values was first incorporated in 1964. By 1966 the techniques used in the analysis had been refined considerably and procedures developed were adopted nationally. The term Cost Benefit Analysis was first applied to the reports in 1967, although they were still strictly project appraisals at that time. Sensitivity analysis of uncertain parameters such as product prices, scheme capital costs and project life were included in the analysis of some schemes.

With the formation of the Economics Division in the latter half of 1970, a Resource Section, within the Division was set up in Palmerston North. Its major responsibility was to investigate and prepare economic reports evaluating agricultural projects from the national point of view. Thus for the first time project evaluation reports originated from the one centre. One result was that reports became consistent in analysis and layout.

In 1971, by directive of Treasury, a ten percent weighting for overseas funds content was added to benefits and costs.

Also at this time the PERT-Beta technique for risk analysis was introduced into the analysis. Subsequently this aspect has been questioned both on theoretical issues and from the practical conclusions that were being drawn from it. The technique was dropped in 1972, and this study grew out of a desire to introduce a satisfactory risk analysis technique for project evaluation work carried out by the Section.

1.3 OUTLINE OF THE STUDY

The study proceeds in the following way. In Chapter II a case is presented for carrying out risk analysis in the public sector, present evaluation techniques are critically examined and it is concluded that techniques which incorporate probability analysis show the most promise for improving evaluation techniques. Chapter III outlines in some detail Analytical Techniques for risk analysis which are based on probability calculus and subjective estimating procedures. Chapter IV introduces the Monte Carlo Simulation technique for risk analysis which although having several advantages over analytical techniques also has major limitations to implementation. In Chapter V a particular project is analysed using the previously discussed traditional methods, analytical techniques and Monte Carlo methods. Chapter VI presents the conclusions to the study and recommendations as to the application of risk evaluation techniques to capital projects in the agricultural sector.

CHAPTER II

THE EVALUATION OF RISK AND UNCERTAINTY IN THE PUBLIC SECTOR: A REVIEW OF LITERATURE

2.1 INTRODUCTION

This chapter proceeds as follows:

1. The concepts of Cost Benefit Analysis and risk and uncertainty are introduced. 2. The relevance of undertaking risk and uncertainty analysis in the evaluation of investment projects in the public sector is discussed and justified. 3. Traditional methods of handling risk and uncertainty in project evaluation are reviewed. 4. The methods used by the Economics Division M.A.F. are reviewed and evaluated. 5. The decision theory approach to risk and uncertainty is discussed, and the concepts of utility analysis and the maximisation of expected utility are introduced.

2.2 COST BENEFIT ANALYSIS AND RISK AND UNCERTAINTY

The basic idea behind Cost Benefit Analysis is to decide whether a project involving public expenditure has a positive or negative effect on society. This implies the evaluation of all the relevant costs and benefits. It involves the welding together of a variety of traditional areas of economic study. These are welfare economics, public finance and resource economics.

Although the origins of Cost Benefit Analysis go back to Dupuit (1844) the practical application of the theory to public investment did not occur until the 1950's. Since then there has been a flood of literature on the theoretical and practical problems associated with the technique. Two widely acknowledged general surveys of the literature are those of Prest and Turvey (1965) and Henderson (1961). Welfare aspects are well covered in Krutilla (1961), and Winch (1971), while a review of investment analysis in the public setting may be found in Muthoo (1972) with applications in Turvey (1968). Practical problems and applications are highlighted in Mishan (1971) and Dasgupta and Pearce (1972). Also, Little and Mirrlees (1969, 1974) have made a major contribution to social cost benefit analysis in developing countries, particularly in the field of shadow pricing.

Major problem areas in Cost Benefit Analysis include the measurement of costs and benefits, external effects, the choice of investment criteria and the measurement of uncertainty. As previously indicated this thesis addresses itself only to the latter problem: that of risk and uncertainty. However, before delving into this, it is important that the terms risk and uncertainty be defined for the purpose of this thesis.

2.2.1 Definitions

Henderson (1968, p 138) writes as follows:

"Risk is defined as a situation in which the outcome is uncertain, but the probability distribution of outcomes is known with certainty. This may be called actuarial risk[2].

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2 see Beard *et al* (1969) for basic text on this subject.

At the other extreme, where nothing whatever is known about the probability distribution we have the case of pure uncertainty. Although most actual situations fall strictly under neither heading, they may quite often approximate more or less closely to one or the other, and be treated accordingly. The intermediate cases which are not close to one extreme or the other can be described as situations of uncertainty, with no qualifying adjective. The degree of actuarial risk may be represented by the variance or the standard deviation of the probability distribution."

"Risky choice prevails when a decision maker has to choose between alternatives, some or all of which have consequences that are not certain and can only be described in terms of a probability distribution" (Dillon, 1971, p 4).

From these two quotations it can be seen that Henderson's "uncertainty" is synonymous with Dillon's "risky choice". In this thesis the terms risk and uncertainty will be used to describe random variables that fall within the bounds of Dillon's "risky choice". That is, random variables that may be described in terms of probability distributions.

2.3 UNCERTAINTY AND THE EVALUATION OF PUBLIC INVESTMENT DECISIONS

In the private sector managers behave as risk averters and generally use interest rates that are much higher than would be the case if costs and returns were known with certainty. The question to be answered here is whether decision makers in the public sector should follow suit and make allowance for risk and uncertainty in public investment projects. Current practice in New Zealand is to make allowances for risk and uncertainty. Battersby and Smallbone (1970) considered that projects in the public sector are often faced with the same uncertainties as those in the private sector and therefore the discount rate should reflect this. They did however "...acknowledge the fact that by and large Government investment is relatively low risk." In recommending a ten percent discount rate they took both time preference and uncertainty into account. The rate is based on the opportunity cost of capital in the private sector, but is reduced slightly to acknowledge the lower risk of Government projects.[3]

This view that risk and uncertainty should be taken into account in the public sector is supported by Hirshleifer (1965, 1966). He argues that the same rate should be used in the private sector - to do otherwise would be an argument for the "second best" (Hirshleifer, 1966, p270). In other words, if a lower interest rate was used in the public sector, projects would be accepted that would otherwise have been rejected by the private sector. This would result in an inefficient use of capital.

On the other hand Arrow and Lind (1970) reject this view. They show that when the risks associated with a public investment are publicly borne, the total cost of risk-bearing is insignificant and therefore Government should

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[3] This was not their only argument for a lower discount rate.

ignore uncertainty in evaluating public investments. Similarly, the choice of discount rate should be independent of considerations of risk. This conclusion is reached, not because Government can pool investments, but because Government can distribute the risk among a large number of people. Their conclusions are conditional on the investment being independent of other components of national income. However, if some Government investments are interrelated they should be evaluated as a "package". After such grouping they contended that there would still be a relatively large number of essentially independent projects.

However, there are several cases where Arrow and Lind's general conclusions do not hold.

The first concerns the situation where a substantial proportion of the benefits and costs accrue directly to individuals. In this case, as the individuals incur the attendant cost of risk taking, it is appropriate to take risk into account.

An example of such a situation could be an irrigation scheme or other land betterment proposal where the farmer bears a major proportion of the costs and benefits. The major sources of uncertainty to the farmer may be listed under the following headings:

(a) Technical This includes such factors as the design failure of the project, the failure to reach estimated physical production levels and the performance of new crops.

(b) Economic The main uncertainties are the availability of finance and the levels of input and output prices.

(c) Personal These factors are mainly psychological. The farmer is concerned with problems such as his own ability to adapt to change, his life expectancy and the ages and attitudes of his children.[4]

However, all these uncertainties will not accrue solely to the project. Past experience has shown that Government is willing to come to the aid of farmers under adverse technical and economic conditions (Frampton, 1971, pp 5-10).

From the farmers' point of view the uncertainty due to the project is almost entirely personal. However, technical and economic uncertainties are present whether the project is implemented or not and these can also be modified by the Government's actions. To the farmer, the uncertainty as to his own performance is paramount. This is borne out by the reluctance of many farmers to vote in favour of schemes even though the monetary returns appear very favourable. They are uncertain as to whether they can cope.

Frengley (1972) points out that these social consequences are rarely considered. The analyses generally assume that an increase in monetary benefits leads to an increase in the welfare of the individual. However, the disutility of adjusting to a new system, of the probable loss of leisure time and the additional worry due to increased capital and labour requirements are major considerations to the farmer.

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[4] This concern with the ability to cope with change is by no means peculiar to farmers. Schon (1971) maintains it is an increasing phenomenon of our times.

It is important that the analyst be aware of these particular uncertainties and modify his expectations of the farmers' future actions accordingly.

The second case where a risk analysis should be carried on a public project occurs when regional considerations are important. For example, the failure of a large project could have particularly damaging effects within a region.

The third case for a risk analysis in public investment occurs when a particular project or package of projects (as defined earlier) is large when compared with national income. This is not such a relevant argument in New Zealand's situation but is appropriate where a small country depends solely on a single large investment. However, of considerable importance to New Zealand is the generally wide fluctuations in the balance of payments. It is a widely held view that attempts should be made to reduce these fluctuations. This implies selecting investment projects that take into account the variance of Present Value where overseas funds are involved. This is particularly important to agriculture as over 70 percent of New Zealand's export receipts still come from meat, wool and dairy produce.

A final reason for taking risk and uncertainty into account stems from the political nature of decision making. Politicians generally have the final say in choosing large scale investment projects, and because they do not wish to be associated with failure, it is likely that the risk and uncertainty aspects of projects play an important role in their decision making.

It appears from the above discussion that there is still a case for evaluating Government projects in the agricultural sector for risk and uncertainty. Briefly, main reasons are that individuals often bear a large proportion of the costs and benefits of agricultural projects, and that regional and political considerations can be important.

The question now remains of how the analysis should be carried out. It is my contention that present methods of dealing with risk and uncertainty in the New Zealand agricultural sector are somewhat arbitrary, generally inconsistent and at best confusing. These methods will now be reviewed and practical alternatives suggested and evaluated.

2.4 TRADITIONAL METHODS FOR HANDLING RISK AND UNCERTAINTY

Traditional methods have generally fallen short of their objectives. This section briefly reviews some of the more common traditional methods and shows where they are deficient. The basis of the problem lies with the data estimation. It is this point which is taken up first.

2.4.1 Data Estimation

The estimation of future costs and revenues are the most difficult aspects of project evaluation. Techniques used for estimation are very diverse. They range from being carefully considered computations to off-hand approximations, little better than guess work.

The basic problem is that most prospective projects for which estimations are to be made are unique. We are not dealing with a situation in which a large body of relevant data allows us to predict the degree of future uncertainty. There is seldom relevant data that may be used directly. Thus the estimates or forecasts are often subjective evaluations based on incomplete evidence indicating what the future might hold.

The methods used depend on the data handling facilities, analysis skills, types of information, and the time and money available for the estimation. A review of estimation procedures in common use is given in Chapter 3 under Forecasting Techniques.

Having estimated values for the elements in the analysis, the problem now arises as to how they should be combined to evaluate a project. Klausner (1968) provides a very good review of the traditional techniques. These include:

2.4.2. The Assumed Certainty Approach

Assumed Certainty is the most common method used in project evaluation. Single estimates are used to determine the outcome of the investment. These best estimates are generally modal or most likely values and therefore when combined do not give the expected value of the project[5]. The major advantage of this approach is that it does not require the analyst to make detailed estimates of cash flows. Indeed the popularity of discounting procedures stems from this fact. However, by itself the assumed certainty approach is not very helpful as it does not explicitly consider the uncertainty of the investment.

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[5] Wagner (1969, p655) shows that given a non-linear function $f(x_1 \ldots x_n)$ of random variables $x_1 \ldots x_n$ it is usually erroneous to assume that $E\left[f(x_1 \ldots x_n)\right] = f\ (E\left[x_1\right] \ldots F\left[x_n\right])$. This is the fallacy of using averages.

2.4.3 Payback Period

Use of the payback period method implies a desire for early rather than late returns. Its major drawback is that it does not take into account the effect of cash flow patterns and length of project life. For these reasons it must be discarded.

2.4.4 Conservative Adjustments to Data

The conservative adjustment approach alters costs up and benefits down to allow for uncertainty. The main problems that occur with this method are that adjustments are entirely subjective, possibly inconsistent and when combined statistically unsound. It becomes very difficult for the decision maker to interpret the adjustments. There is also a very small likelihood that all the conservative estimates will occur together. This leads to what Klausner calls an "overkill" effect. At best, this obscures the investment picture and at worst gives an entirely false impression.

2.4.5 Sensitivity Analysis

The sensitivity analysis method usually consists of varying within a range, on a percentage basis, the uncertain elements, one at a time. It has the good effect of highlighting the relative importance of accurately estimating each element. For this reason it may be useful in a post analysis stage to isolate one or two critical elements. However, as a method of analysis by itself it lacks conciseness, and comprehensiveness. A major problem arises because it does not usually give an estimate of the probability that a percentage shift in an element may occur. However, if the probability distribution of a variable is considered in deciding the range within which the sensitivity analysis is to be conducted, then the more sophisticated method to be discussed later can be applied.

2.4.6 Risk Adjusted Discount Rate

Raising the minimum acceptable rate of return is basically the same as using the assumed certainty method except that it adjusts the discount rate up as the risk and uncertainty of a project increases. As with the payback period method this technique discriminates against projects with a long life. Its major drawback however, is that it becomes subjective in deciding what level the discount rate should be raised or lowered for different levels of risk between projects. The arguments for using this method are essentially circular. It is assumed that the riskiness of the project is known so that it can be classed with similar ventures which pay a known market price for capital. As Van Horne (1972), and others point out it is the risk-free rate (the time value of money) that should be used as the discount rate.

2.4.7 Running Multiple Cases

In running multiple cases several or all the elements are changed at once. The most usual case is evaluating the project with all elements first at optimistic levels, then at most likely levels and finally at pessimistic levels. However, knowing the extreme range that the project can take does not provide the decision maker with much useful information unless he knows the probability of occurrence. Even then the probability of all the values being high or low will be extremely small. This method is a variant of sensitivity analysis.

2.5 THE METHODS USED IN NEW ZEALAND AGRICULTURE

2.5.1 Cost Benefit Analysis

Ward (1964) was the first to advocate the use of Cost Benefit Analysis in New Zealand. During the mid 1960's New Zealand had just emerged from a period of slow growth in the 1950's and early 1960's. The prevailing mood was orientated towards economic growth and Ward saw the need for a systematic method for the evaluation of the large number of investment projects set before Government. His concern was that there should be a common basis of comparison (between project reports) so that the relative merits of individual projects could be reviewed in a more consistently objective manner[6].

Since then the technique of project evaluation has been disseminated through the universities[7] to Government and Local Body Agencies which are largely responsible for implementing the technique.

2.5.2 The Problem of Uncertainty

Traditionally project evaluation procedures have failed to deal adequately with the risk and uncertainty problem.

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[6] The Maraetai Study (Ward *et al.* 1965) served as an illustration of the suggested methodology.

[7] Jensen (1968) edited the proceedings of a seminar designed to thrash out a general consensus on methods of analysis and terminology for the New Zealand scene.

In a review of agricultural and forestry evaluation studies in New Zealand Orsman and Johnson (1973) found that only one report included a full scale analysis of uncertainty.[8] Most authors approached the problem in a qualitative way. Estimation of data proved to be the most difficult problem. For example Chisholm (1962) emphasised the difficulties of estimating physical input-output relationships, and predicting resource costs and product prices up to 50 years ahead.

The methods used for dealing with risk and uncertainty may be summarised under the following headings:

2.5.3 The Methods

(a) Conservative Adjustments to Data. Generally when conservative adjustments to data are made, cost type elements are adjusted up and revenue type elements are adjusted down (Twomey, 1955).

(b) Sensitivity Analysis. Where it is felt that a particular element has a critical effect on the profitability of a project then a sensitivity analysis is carried out. There are many examples including:- capital costs (van Asch, 1970), project life (Butler, 1969), area developed (Bryant, 1972) and the discount rate (Plunket, 1964).

(c) Multiple Cases. The usual situation is to analyse the project at three levels using pessimistic, most likely and optimistic estimates of elements. This method forms the basis of the present technique used by the Resource Economic Section of the Ministry of Agriculture (Forbes, 1973).

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8 Wise Land Use and Community Development 1970.

(d) Risk Adjusted Discount Rate. The ten percent discount rate used by all Government Departments in New Zealand is risk adjusted. In a Treasury paper Battersby and Smallbone (1970) stated that "...Public investment projects are often subject to the same uncertainties as are projects in the private sector and the discount rate used should reflect this fact." The authors considered that the "risk free" rate was about 6.5 percent. This was based on the Cost of Borrowing approach. However, in their view, the Opportunity Cost of Capital (12-15 percent) provided a more acceptable basis for choosing the rate. The ten percent rate finally suggested, was a value judgment, in part to "acknowledge the fact that by and large Government investment is relatively low risk."... that is, lower than the risks in the private sector.

(e) Combinations of Above Methods. Often aspects of the above four methods will be incorporated in the one analysis. The result being that decision makers are not faced with a consistent set of analyses of different projects.

To use a risk adjusted discount rate and then allow for risk through the use of variance is double counting and therefore tends to reject more projects than is appropriate.

Also risk adjusted discount rates imply that all variables in the present value equation are subject to the same degree of risk, which is proportionate to time - an invalid argument.

(f) An Exception to the Rule. There has been one detailed analysis of uncertainty in project evaluation in New Zealand. It appears in "Wise Land Use and Community

Development" (1970), a Ministry of Works Publication. In Appendix XI of the report Vignaux describes the uncertainty analysis. It is based on the concept of Range (Vignaux, 1966). Range gives the simplest estimate of the standard deviation for normal populations where Range equals the High minus the Low estimate (Snedecor, 1961). A major assumption in the analysis is that the ranges of different variables are independently normal. Given this assumption Vignaux calculates the total uncertainty by taking the square root of the sum of squared Ranges. Probability statements are then made by an appeal to the Central Limit Theorem.

It is stated in the report that the major uncertainty lies in price trends for Agriculture and Forestry. Thus, for prices the Range is calculated by taking an upward trend of 0.5 percent per year and an equal downward trend. Without giving a detailed criticism of the analysis it is fair to say that the analysis contains heroic assumptions about the independence of variables. There is also a conservative bias in a number of the estimates and the mathematics and use of Range to estimate the standard deviation is open to question (Snedecor, 1961, p 110). However, to my knowledge, the report represents the first serious attempt at a full uncertainty analysis in New Zealand and must be commended for this.

2.5.4 <u>Deficiencies of Traditional Methods.</u>

Klausner op.cit. ably sums up the deficiencies of these traditional methods.

"a. Judgment is usually applied to the results of the analysis rather than to the underlying assumptions.

b. There is no overall indicator of outcome variability generated.

c. There is no accounting for investment element interaction.

d. These methods result in a qualitative "feel" rather than a quantitative assessment of risk.

e. They do not provide a consistent analysis framework – either project to project or individual to individual."

In an attempt to overcome these types of criticism Shepherd (1970) introduced the so-called PERT – Beta[9] method for handling risk and uncertainty into the evaluation of projects carried out by the Resource Section.

2.5.5 The PERT – Beta Method

The procedure involves the calculation of the mean and standard deviation of the project's present value. This is carried out as follows:

Firstly the project is analysed at three separate levels of prices.[10] These prices correspond to pessimistic, most likely and optimistic levels. It is assumed that there is only a 0.05 probability of the pessimistic or optimistic levels being exceeded.

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[9] PERT – Program Evaluation and Review Technique. See Malcolm et al. (1959) for details of the technique, Moder and Rodgers (1968) for practical applications and the Federal Electric Corp. (1967) for a programmed introduction to the method.

[10] Uncertainty is evaluated for product prices only.

The three net cash flows(pessimistic, most likely and optimistic) are then discounted separately using a ten percent social discount rate and the expected present value and standard deviation are calculated as follows:

$$E(PV) = \frac{a + 4m + b}{b} \quad \ldots\ldots\ldots\ldots (1)$$

$$S.D. = \frac{b - a}{6} \quad \ldots\ldots\ldots\ldots (2)$$

where a = the present value of the pessimistic net cash flow.

m = the present value of the most likely net cash flow.

b = the present value of the optimistic net cash flow.

Equation (2) infers that the range given by the optimistic minus the pessimistic net cash flows covers 99 percent of the total range (or six standard deviations). Dividing the range by six gives the value for one standard deviation.

Thus, assuming that the present value is approximately normally distributed, equation (2) may be interpreted as stating that there is a 63 percent chance that the actual present value will lie within plus or minus one standard deviation of the calculated mean.

However, use of equation (2) relies on a very dubious assumption; namely that the prices used in the calculation are fully dependent. That is, they have a correlation coefficient of one. This means that if one price is at its optimistic level then all other prices are assumed to be at optimistic levels - and moreover for the whole of the project life.

If, on the other hand, it is assumed that all the prices are strictly independent, then equation (2) becomes an invalid estimate of the standard deviation. For example, if there are ten independent variables in the project each with a probability of 0.05 of being optimistic or pessimistic, then the probability that they would all be optimistic is $(0.05)^{10}$. This is 10^{-14}. The range that this covers would be approximately 20 standard deviations. Therefore dividing the range by 20 would give a more meaningful estimate of the standard deviation.[11] Equation (2) would then become:-

$$S.D. = \frac{b - a}{20} \qquad \ldots\ldots\ldots\ldots (3)$$

Use of equation (3) as a valid criterion for judging the uncertainty of projects rests on the assumptions that prices are strictly independent and that they remain constant over the life of the project.

As these assumptions are not acceptable the PERT - Beta Technique was discarded in 1972.

2.5.6 A Retreat to Traditional Methods

The method now used by the Resource Economics Section is a less positive explanation of the risk and uncertainty associated with projects. It is based on the three product price levels which give a range or space wherein the results may lie. It is left to the decision maker to define at what point within the range the result should lie (Orsman and Johnson, 1973 pp 22-23). However, the analyses still incorporate all the dubious points of the traditional methods.

The techniques may incorporate biased collection of data. The uncertainty issue is still confused by the use of different combinations of sensitivity analysis. Double

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11 This was pointed out by Le Page (1973) in an Economics Division internal discussion paper.

counting for risk and uncertainty is present due to the risk adjusted discount rate and the use of conservative data adjustments. Finally no probability estimate is placed on the range of the present value. While these inconsistencies are present it cannot be hoped to make good decisions when comparing a number of projects or in fact judging whether a project passes a certain profitability criterion.

In order to eliminate the above inconsistencies and deficiencies a return must be made to basic theoretical decision making. In this next section the decision theory approach to risk and uncertainty is discussed. It will be found to lead to utility analysis and the maximisation of expected utility.

2.6 THEORETICAL CONSIDERATIONS

2.6.1 The Decision Theory Approach

This approach assumes that a decision maker is faced with a number of alternative decisions (D_1, D_2, ..., D_m) and a number of possible states of the world (S_1, S_2, ..., S_n). The Ds and Ss form mutually exclusive and exhaustive sets so that only one state will prevail (although which one is not known), and a decision must be chosen from the list given. For each D_i and S_j a "payoff" a_{ij} exists and can be calculated. The a_{ij}s are arranged in a payoff matrix where the rows correspond to decisions and the columns to states of the world. No assumption is made about the relative likelihood of the various states occurring. That is, pure uncertainty exists.

The problem is then to find a criterion for selecting the best decision. A number of criteria have been proposed in the literature. The most important are listed below.

(a) The Maximin Pay-off (Wald) Criterion

(b) The Minimax Regret Criterion (Savage).

(c) The Index of Pessimism Criterion (Hurwitz).

(d) The Principle of Insufficient Reason (Laplace, Bayes).[12]

These decision criteria may be classified on the information they take into account. The first two only allow for the worst outcomes, the third uses the worst and the best outcomes only, while the fourth assumes that all outcomes should be taken into account. All have been found to give illogical choices under certain circumstances (Dorfman, 1962).

From the practical point of view Mishan (1971, pp 298-299) shows that even for a very simple cost-benefit problem involving a four year stream of net benefits, with four items taken at three levels in the first year and five levels in the second to fourth years, the number of permutations in the payoff matrix reaches close to 20 billion. It would obviously be impractical to calculate and evaluate a system of this size, let alone a realistically sized problem.[13]

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[12] See Dasgupta and Pearce op.cit. for a description, example and application of each criterion.

[13] The Expected Value of Perfect Information (EVPI) or Expected Opportunity Loss (EOL) principle suffers the same defect - see Canada (1971, p 294).

The above criteria were formulated with the assumption that because the states of the world could not be objectively specified probabilities should not be applied to them. Savage (1954) led a school of thought which rejected the objectivity requirement. He showed that a decision maker's belief about the relative likelihood of an event could be incorporated into a subjective probability distribution that had the same properties as an objective distribution. His argument stressed that it was an improvement to include all the information available in an explicit way rather than ignore it as "vague opinion".

On removing the objectivity condition the subjective weightings can be applied to the Bayes procedure. The technique is to calculate the expected value of each decision then choose the decision with the highest expected value as the criterion.

The expected value of each decision is given by the first moment of its probability distribution (μ_1) where

$$\mu_1 = \sum_i P_i \ X_i \quad \ldots\ldots\ldots\ldots\ldots (4)$$

and X_i equals the value of the P_i possible outcome and the probability of its occurrence.

Decisions are then based only on the first moment, but to fully describe a probability distribution all n moments must be taken into account. The n^{th} moment is given by :-

$$\mu_n = \sum_i P_i \ (X_i - \mu_1)^n \ldots\ldots\ldots (5)$$

It is generally recognised, though, that the first two moments contain most of the essential information about a distribution.

The second moment (μ_2), also called the variance, describes the degree of uncertainty or the spread of possible outcomes around the expected value (μ_1). Sometimes the third moment, which measures the asymmetry of the distribution may be considered.

When only the first moment of the distribution is used as a choice indicator it can be shown that under certain circumstances this can lead to illogical choice (Adelson, 1965, p 30). In an attempt to resolve the problem, procedures which use the first two moments were developed. The most promising of these techniques is Utility Analysis as it incorporates both expectations and risk and uncertainty into a single criterion.

2.6.2 Utility Analysis

Utility analysis has an ancient history dating back at least to Bernoulli (1738). However, modern utility analysis has been attributed to von Neumann and Morgenstern (1944) although Friedman and Savage (1948), Savage (1954) and Schlaifer (1959) are among those who contributed to its development. In later years Dillon (1971) provides an excellent expository review of the whole subject.

The central idea is that choices among alternatives involving risk can be explained by the maximisation of expected utility. This utility is a hypothetical quantity related to the expected cash value but including a weighting for the risk or likely spread about the expected cash value. It is postulated that a decision maker attempts to maximise his expected utility rather than just the cash value.

The relative value to the decision maker, of different levels of gains or losses can be expressed by his utility function. Several methods have been suggested to derive utility functions, of which the modified von Neumann Morgenstern method appears to be satisfactory (Dillon, 1971 p 25). This method is based on the certainty-equivalent concept. The decision maker is asked a series of questions about his preference for certain income against a 50:50 probability of some larger income. A function can then be plotted and the functional form fitted by standard estimating techniques such as regression analysis. One suggested functional form is the quadratic. Typical utility functions, of this type, for a conservative or risk averse decision maker are shown in Fig.1 (a) and (b). (Officer et al. 1967, p 173).

Figure I <u>Utility Functions</u>

Fig. 1 (a)

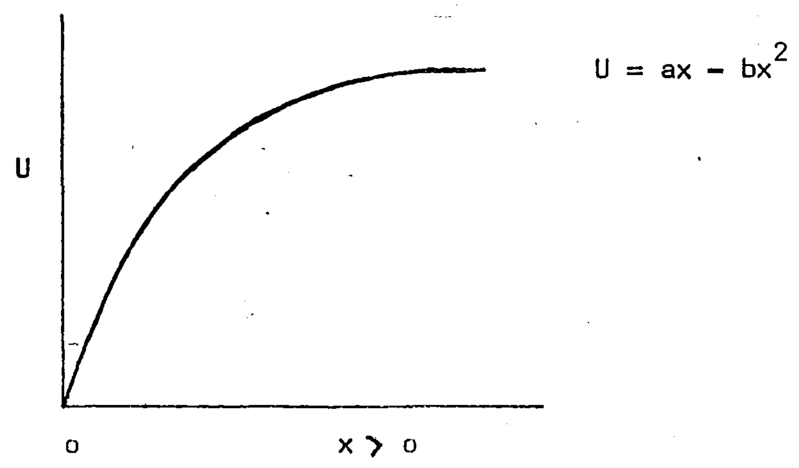


Fig. 1 (b)

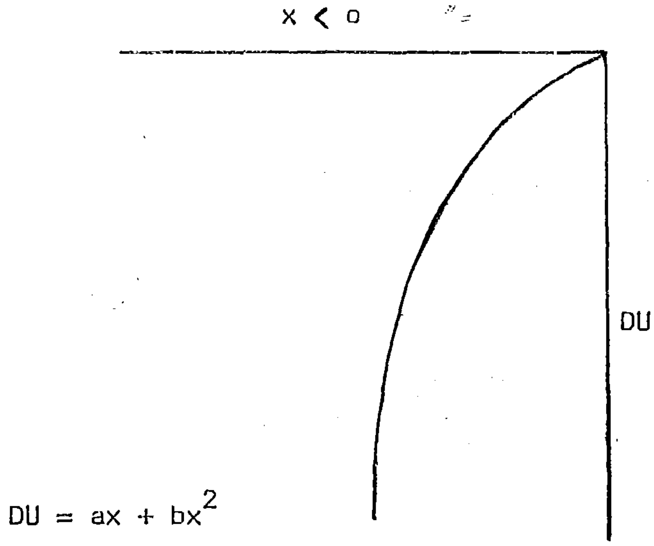


Fig I (a) shows diminishing marginal utility for increasing returns and Fig I (b) shows increasing marginal disutility for increasing costs.

In practical terms the utility function can be expressed in terms of the expected value and variance of investment projects where the function is assumed to be quadratic. This is represented by the first two moments of the probability distribution of the present value.

The quadratic utility function may be derived as follows.

$$\text{given } U = ax + bx^2 \quad \ldots\ldots\ldots\ldots \quad (6)$$

then taking expectations gives -

$$E\,(U) = E\,(ax + bx^2) \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$\text{then because } Var\,(x) = E(x^2) + \left[E(x)\right]^2 \quad \ldots \quad (8)$$

$$E\,(u) = aE(x) + b\,Var\,(x) + b\left[E(x)\right]^2 \quad \ldots \quad (9)$$

where U = utility

x = present value

a, b = constants

Equation (9) represents a ranking criterion expressed in terms of the expectation and variance of present value.

This appears to be a major improvement over traditional methods. Unfortunately the concept of a utility function as a proxy for the wellbeing of society is rejected by most writers. Mishan (1971, p 296) states "it would seem quite impractical to derive such a curve uniquely for a large number of people; and for society in general quite impossible."

The expected present value of a project is not the gain of a single person. It is the result of the aggregate gains and losses of all people involved in the project. To replace expected present value by total utility, the utilities of all people affected by the project have to be added together by some arbitrary weighting system. This is clearly impossible. Also, Arrow (1971, p 102) shows that use of quadratic functions can lead to paradoxical results. The quadratic function implies that as an individual (or a nation) becomes wealthier, the amount of risky investment would tend to diminish. From both a practical and intuitional point of view this appears to be incorrect.

2.6.3 Practical Implications

Dasgupta and Pearce (1972, p 186) state that as yet there is no generally accepted method for dealing with risk aversion in cost benefit analysis in a systematic way. They suggest that the only feasible alternative is to estimate the expected social utility and reduce it by a certain percentage depending on the extent of risk aversion thought to be appropriate.

Little and Mirrlees (1974, p 319) suggest the use of the certainty equivalent approach where the deduction to be made from the expected present value on the account of uncertainty should be the variance of the project divided by the gross national product of the country.

Another slightly different approach as suggested by Fromm and Taubman (1968) is to present the policy making group with a series of assessments based on a variety of utility functions, leaving the policy making group to make its own choice of utility function. Dillon (1971) reports that although this sounds far-fetched, similar procedures already operate in the Netherlands and some East European countries.

There is one point that remains clear amid the murk of the risk aversion problem. This is that all projects should be evaluated in a systematic way at the risk-free rate of interest. If it is desired to take account of risk and uncertainty it should be done in conjunction with the expected present value calculated at that rate.

One such way is to calculate the variance of the present value. Programming methods will now be developed to do this.

2.7 SUMMARY

A case has been put for the evaluation of risk and uncertainty in agricultural projects, but on a review of evaluation techniques in use it has been found that many deficiencies exist. In a search for a practical solution to the risk analysis problem we have seen that decision theory shows very little promise. The concept of utility as a single measure of the well-being of society, while being intrinsically neat, is impossible to derive at the national level. Rather the solution is to make the risk and uncertainty explicit by presenting the present value in terms of its probability distribution - leaving it to the decision maker to weight the expected outcome by its variance as he sees fit. It should be the analyst's aim to present all the available and relevant information to the decision maker in an unbiased and explicit way. In the next two chapters mathematical programming methods are developed to achieve this aim. The first method is the Analytical Technique.

CHAPTER 3

THE ANALYTICAL METHOD

This chapter outlines the steps involved in the analytical method of evaluating risk and uncertainty in project appraisal. Firstly it is shown how the mathematical model was developed through the use of probability calculus. Then the following aspects of the model are discussed: Forecasting methods, the subjective probability distribution, correlation, and lastly the problems associated with the use of the model.

3.1 DEVELOPMENT OF THE MATHEMATICAL MODEL

Hillier (1963) laid the foundations of risk analysis. His aim was to show how, under certain circumstances, such information as the probability distribution of the internal rate of return, present worth or annual cost of a proposed investment could be derived. The approach used was similar to that expounded by Markowitz (1959) to determine the portfolio of market shares to provide the minimal variance of rate of return for a given expected rate of return. Hillier followed Markowitz by using the mathematics of combining random variables to estimate the expected present value and variance of present value of a project, given the mean and standard deviation for each cash flow and the cross correlations between the flows.

3.1.1 Hillier's Formulation

The investment considered is one which would result in cash flows over a number of years (where cash flows are defined as money payments to or from a project).

Let X_t be the random variable that takes the value of the net cash flow in the t^{th} year, where $t = 0, 1, 2, \ldots, n$, assume that X_t has a normal distribution with known mean μ_t and standard deviation σ_t. Assume also that the relationships between the X_t for different t is either one of mutual independence or complete correlation. This forms a restrictive model which is rather unrealistic but it serves as a starting point.

(a) Probability Distribution of the Present Value.

The present value of the cash flow in year t (P_t) is given by

$$P_t = \frac{X_t}{(1 + i)^t} \qquad \ldots\ldots\ldots\ldots (10)$$

where i equals the discount rate. It follows that the expected present value is given by :-

$$E(P) = \sum_{t=o}^{n} \frac{E(X_t)}{(1 + i)^t} \qquad \ldots\ldots\ldots\ldots (11)$$

and if the $X_o, X_1, \ldots, X_n$ are mutually independent then the variance of the present value is given by:-

$$\text{Var } (P) = \sum_{t=o}^{n} \frac{\text{Var } X_t}{(1+i)^{2t}} \qquad \ldots\ldots\ldots\ldots (12)$$

where the $X_o, X_1 \ldots, X_n$ are perfectly correlated the variance becomes

$$\mathrm{Var}\ (P) = \left[\sum_{t=o}^{n} \frac{\sigma_t}{(1+i)^t} \right]^2 \quad \ldots\ldots (13)$$

In the case where there are both independent flows (Y_t) and correlated flows (Z_o^k, Z_1^k, ..., Z_n^k) for $k = 1,2,\ldots,m$, then the expected present value is

$$E(P) = \sum_{t=o}^{n} \frac{E(Y_t) + \sum_{k=1}^{m} E(Z_t^k)}{(1+i)^t} \quad \ldots\ldots (14)$$

with variance.

$$\mathrm{Var}\ (P) = \sum_{t=o}^{n} \frac{\mathrm{Var}(Y_t)}{(1+i)^{2t}} + \sum_{k=1}^{m} \left[\sum_{t=o}^{n} \left(\frac{\mathrm{Var}(Z_t^k)^{\frac{1}{2}}}{(1+i)^t} \right) \right]^2 \quad (15)$$

In equations (14) and (15) it is assumed that the Z_t^k cash flows are perfectly correlated with the corresponding cash flows in other periods. For any given year, however, the cash flows are independent one with another.

Thus Hillier considers three cases:-

(i) independence between successive periods

(ii) perfect correlation between periods

(iii) a net cash flow that is made up of an independent cash flow, plus m cash flows that are correlated between periods (autocorrelated) but independent within a period.

(b) The Central Limit Theorem. By reference to the central limit theorem it can be concluded that the probability distribution of the present value will be approximately normal (Mood and Graybill, pp 149-153). This allows the use of the normal tables to calculate the probability of obtaining any given value, for example the probability that $P < 0$.

(c) The Probability Distribution of IRR. The Internal Rate of Return (R) is defined as that value of i for which $P = 0$. The projects accepted are those where $R > i$. There is a certain amount of controversy regarding the acceptability of the IRR for general use in investment appraisal (Hawkins and Pearce 1971 pp 29–38) but as the concern here is with methodology it will be assumed that the measure is acceptable.

The procedure for finding the probability distribution of the IRR is straightforward. An arbitrary value of i is selected and the probability distribution of P is calculated as described above. The next step is to find the probability that $P < 0$, then this is the probability that $R < i$, i.e.

$$\text{Prob}\left\{R < i\right\} = \text{Prob}\left\{P < 0 \mid i\right\} \quad \ldots\ldots (16)$$

To find the cumulative probability distribution of R, the calculation of Prob $\left\{P < 0 \mid i\right\}$ should be repeated for as many values of i as desired.

Hillier (1965) pointed out that equation (16) "...should be viewed primarily as a computing equation for practical application which is almost exact only when the IRR is a valid criterion with probability essentially one."

Bernhard (1967) shows that the internal rate of return criterion is not valid in general. One of the major problems is that of multiple real roots. deFaro (1974) suggests an algorithm for implementing the IRR criterion which formally takes this problem into account. Another problem is that the project's return is not independent of the cost of capital and therefore the IRR criterion could give ambiguous and incorrect results (Teichroew et al. 1965). Herbst (1974) has derived a FORTRAN program that takes this interdependence between project returns and the cost of capital into account.

3.1.2 A More Practical Formulation

Wagle (1967) extended Hillier's analysis by discussing methods to handle the cases where the means and variances of the different cash flows were not known directly, but the means and variances of the factors which make up the cash flows are known.

The net cash flow is derived from a number of distinct sources. For example, the net income from a project could be a function of:- capital costs, operating costs, and revenue which may be derived from several sources. To allow for the relationships that occur between these sub cash flows they must be treated separately.

(a) The Mean and Variance of a Sum of Random Variables.

Let the random variable X_{tj} denote the cash flow in period t from the j^{th} source where t=0,1,2,..., n and j=1,2,...,m. It is assumed that X_{tj} has a finite mean μ_{tj} and variance σ^2_{tj}. Then the net cash flow in period t is given by

$$X_t = X_{t1} + X_{t2} + \ldots X_{tm} \qquad \ldots\ldots\ldots\ldots (17)$$

Thus the expected cash flow in period t is

$$E\ (X_t) = \sum_{j=i}^{m} \mu_{tj} \quad \ldots\ldots\ldots\ldots (18)$$

and the variance of the cash flow in period t is

$$Var\ (X_t) = \sum_{j=1}^{m} \sigma^2_{tj} + 2 \sum_{i \neq j} Cov\ (X_{tj}, X_{ti}). \quad (19)$$

It follows that the expected present value and variance of the present value are

$$E\ (P) = \sum_{t=0}^{n} \left[E(x_t)\alpha^t \right] \qquad \text{where } \alpha = \left(\frac{1}{1+i}\right) \quad .\ .\ (20)$$

$$Var\ (P) = \sum_{t=0}^{n} \left[Var\ (X_t)\alpha^{2t} \right] + \sum_{j=1}^{m} \left[2 \sum_{t \neq t'} Cov\ (X_{tj}, X_{t'j})\alpha^{t+t'} \right] \quad .\ .\ (21)$$

The covariances are functions of the correlation coefficients and standard deviations between the variables. The covariance between different variables within the same period is

$$Cov\ (X_{tj}, X_{ti}) = r_{ij}\ \sigma_{tj}\ \sigma_{ti} \quad .\ .\ .\ .(22)$$

This topic of correlation and dependency will not be elaborated here but left to a later section.

(b) <u>The Variance of a Weighted Sum of Variables.</u>

In some cases the cash flows element X_{tj} may be the product of a weight (A_j) and a random variable (X_j) which has a standard deviation σ_j. It can be shown (Markowitz, 1959, p 94) that when this occurs the variance of the weighted sum of random variables is given by

$$
\begin{aligned}
\text{Var}\ (X_t) \ = \ & A_1^2\ \sigma_1^2 \ + \ A_2^2\ \sigma_2^2 \ + \ \ldots\ldots\ + A_n^2\ \sigma_n^2 \\
& +2\ A_1\ A_2\ \sigma_1\ \sigma_2 \ + \ 2\ A_1\ A_3\ \sigma_1\ \sigma_3 \ + \ \ldots \\
& \ldots + 2\ A_{m-2}\ A_m\ \sigma_{m-2}\ \sigma_m + 2\ A_{m-1}\ A_m\ \sigma_{m-1}\ \sigma_m
\end{aligned}
\qquad \ldots. (23)
$$

When this situation occurs the weights must be incorporated into the variance of present value equation. On incorporating the weights equation (21) becomes

$$
\begin{aligned}
\text{Var}\ (P) \ = \ & \sum_{j=1}^{m} \left[\sum_{t=0}^{n} A_{tj}^2\ \sigma_{tj}^2\ \alpha^{2t} \right] \\
& + \sum_{i}^{p} \sum_{\substack{j \\ i \neq j}}^{m} \left[2r_{ij} \sum_{t=0}^{n} A_{ti} A_{tj}\ \sigma_{ti}\ \sigma_{tj}\ \alpha^{2t} \right] \\
& + \sum_{j=i}^{m} \left[2 \sum_{t=0}^{t=n-1} \sum_{t'=1}^{t'=n} A_{tj}\ A_{t'j}\ r_{tt'}\ \sigma_{tj}\ \sigma_{t'j}\ \alpha^{t+t'} \right]
\end{aligned}
\qquad \ldots. (24)
$$

Murphy (1968) demonstrates the use of this model.

(c) The Variance of a Product of Two Random Variables. In many cases the random variable X_{tj} is itself the product of two random variables (x and y), as for example where both yield and price are random variables, then the mean and variance of the product is given by

$$E(x,y) \approx \mu_x \cdot \mu_y + r_{xy} \sigma_x \sigma_y \quad \ldots \quad (25)$$

$$Var\,(x,y) \approx \mu_y^2 \sigma_x^2 + \mu_x^2 \sigma_y^2 + 2 \mu_x \mu_y r_{xy} \sigma_x \sigma_y \quad \ldots \quad (26)$$

These formulae are approximate for both dependent and independent cases (Springer, et al. 1968). Equation (26) holds when the coefficient of variation ($\frac{\sigma}{\mu}$) is small. For example, if ($\frac{\sigma}{\mu}$) is less than 0.1 then the error will be less than 0.01.

(d) The Variance of the Sum of Products of a Number of Random Variables. Frequently, either or both of the variables in a product of two random variables will be dependent on another variable. A case in point would be where the gross income for wool is the product of two random variables - wool weight and wool price, but the variable cost associated with wool is dependent on the total amount of wool.

To handle this sort of situation we must resort to a generalisation, as the statistical formulae quickly become very complex. The generalisation utilizes the first two terms in the Taylor Series Expansion[14]. Using the Taylor's series the mean and variance of a number of random variables may be determined as follows.

"The mean of a function of n random variables is approximately equal to the function of the means of the variables if the function is approximately linear." For example

$$P = x_1 + x_2 - x_3 + x_4 \cdot x_5 \quad \ldots\ldots\ldots (27)$$

then

$$\mu_P \approx \mu_{x_1} + \mu_{x_2} - \mu_{x_3} + \mu_{x_4}\, \mu_{x_5} \quad \ldots\ldots (28)$$

"The variance of the function is derived by differentiating the equation for μ_P. It is approximately equal to the sum of the products of partial derivatives squared and the variance of each variable, plus two times the sum of the products of partial derivatives, coefficients of correlation, and standard deviations for all combinations of pairs of variables." (Springer, 1968, p 121–122). This is

$$\mathrm{Var}\,(P) \approx \left(\frac{\partial P}{\partial x_1}\right)^2 \sigma^2_{x_1} + \left(\frac{\partial P}{\partial x_2}\right)^2 \sigma^2_{x_2} + \ldots + \left(\frac{\partial P}{\partial x_m}\right)^2 \sigma^2_{x_m}$$

$$+2 \left(\frac{\partial P}{\partial x_1}\right) \left(\frac{\partial P}{\partial x_2}\right) r_{12}\, \sigma_{x_1} \sigma_{x_2}$$

$$+2 \left(\frac{\partial P}{\partial x_1}\right) \left(\frac{\partial P}{\partial x_3}\right) r_{13}\, \sigma_{x_1} \sigma_{x_3} + \ldots$$

$$+2 \left(\frac{\partial P}{\partial x_{m-1}}\right) \left(\frac{\partial P}{\partial x_m}\right) r_{m-1,m}\, \sigma_{x_{m-1}} \sigma_{x_m} \quad \ldots (29)$$

---

[14] See any calculus text such as Maxwell (1962). An Analytical Calculus Vol II.

The total number of combinations of pairs of variables in the variance covariance equation above is $\binom{n}{r}$ which equals $\frac{n!}{r!(n-r)!}$ where n is the number of variables and r is two.

To apply Taylor's approximation to the evaluation of risky or uncertain projects merely replace the A_is of equation (24) with the partial derivatives $\left(\frac{\partial P}{\partial x_i}\right)$ of equation (29).

Let $\left(\frac{\partial P}{\partial x_i}\right)$ equal D_i, then the variance of the present value of the sum of a number of complex combinations of random variables is given by:-

$$\operatorname{Var}(P) \approx \sum_{j=1}^{m} \left[\sum_{t=0}^{n} D_{tj}^{2} \sigma_{tj}^{2} \alpha^{2t} \right]$$

$$+ \sum_{\substack{i \\ i \neq j}}^{p} \sum_{j}^{m} \left[2 r_{ij} \sum_{t=0}^{n} D_{ti} D_{tj} \sigma_{ti} \sigma_{tj} \alpha^{2t} \right]$$

$$+ \sum_{j=1}^{m} \left[2 \sum_{t=0}^{t=n-1} \sum_{t'=1}^{n} D_{tj} D_{t'j} r_{tt'}^{|t-t'|} \sigma_{tj} \sigma_{t'j} \alpha^{t+t'} \right] \qquad \ldots . (30)$$

The logic of the approach can be considered by the following simple model of a project.

Let the net benefit in year t be a function of five random variables:

$$B_t = X_1 (X_2 * X_3 - X_4) - X_5 \qquad \ldots . (31)$$

then the expected present value is given by:

$$E(P) = \sum_{t=0}^{n} \left[X_{1t} \quad (X_{2t} * X_{3t} - X_{4t}) - X_{5t} \right] \alpha^{t} \quad \ldots (32)$$

The variance of the present value can be obtained from equation (30). This requires estimates of the standard deviations (σ_i) coefficients of correlation (r_{ij}) and ($r_{tt'}$) and the partial derivatives (D_i) for all the uncertain variables.

The partial derivatives D_i are found by differentiating the formula for P, treating X_i as a variable and all the other variables as constants. The result is

$$P = X_1 \quad (X_2 * X_3 - X_4) - X_5 \qquad \ldots\ldots (33)$$

thus

$$D_1 = \frac{\partial P}{\partial X_1} = X_2 * X_3$$

$$D_2 = \frac{\partial P}{\partial X_2} = X_1 * X_3$$

$$D_3 = \frac{\partial P}{\partial X_3} = X_1 * X_2 \qquad \ldots\ldots (34)$$

$$D_4 = \frac{\partial P}{\partial X_4} = -X_1$$

$$D_5 = \frac{\partial P}{\partial X_5} = -1$$

The calculated D_i for each year are then substituted straight into equation (30) and the variance calculated.

This now completes the mathematical model, examples of the use of the Taylor's expansion may be found in Canada and Wadsworth (1968), they assume a constant net annual receipt or disbursement through the project life. Springer op. cit. and Smith (1969) give examples in the business situation.

(e) Project Life. If the project life n is a random variable, then the estimates of the mean and variance of present value must be combined to give a distribution over n, for the unconditional mean and variance for the overall present value (Wagle, 1967, pp 16-17).

Tersine and Rudko (1972) developed a model for a Beta distributed project life. They suggest that the approach could be applicable where the project life is highly volitile : for example, the launching of a new product and the associated consumer acceptance. Van Horne (1972) proposed a method for dealing with uncertainty in the project life by the use of conditional probabilities. However, Reutlinger (1970, p 39) notes that "... concern about uncertainty from this source becomes less important when the discount rate is high and the earliest date of termination is quite a long time in the future. If the discount rate is ten percent, for instance, the present value of benefits is hardly affected by whether the life of the project is thirty or fifty years."

(f) Evaluating the Proposal. The expectation and variance of the overall present value provides a basis for measuring the risk and uncertainty involved in the project. Where no assumptions can be made about the shape or type of distribution of the present value then Chebychev's inequality gives the probability that the present value will lie within certain bounds. Chebychev's inequality states:-

$$\text{prob}\left\{ |P - E(P)| \leqslant k\,\sigma_p \right\} \quad \frac{1}{1-k^2} \quad \ldots\ldots \quad (35)$$

where k=3 equation (35) states that the probability of the present value lying within plus or minus three standard deviations of the expected value is greater than 89 percent. However a distribution has to become quite peculiar before the probabilities get close to Chebychev's bounds.

Where the distribution is known to be normal, the bounds are reduced and the probability that an observation lies within plus or minus three standard deviations of the expected value becomes 99.72 percent. In practice, for distributions which are nearly normal, it is highly unlikely that a given observation will lie outside this range. By an appeal to the Central Limit Theorem the normality assumption can usually be made, except for extraordinary situations (Hillier 1969 pp 24-29). Also Hayya and Cunningham (1974) have shown that the Central Limit Theorem still holds where the cash flows are autocorrelated.

Given the expected present value and variance, plus a normality assumption, it is possible to construct a graph (from the normal tables) which shows the measure of risk and uncertainty associated with the project.

First it is necessary to standardise the mean (μ) and standard deviation (σ) of the present value, so that it has a standard normal distribution Z with mean zero and variance one.

$$Z^1 = \frac{P_i - E(P)}{\sigma_p} \qquad i = 1,2 \ldots n \qquad \ldots . (36)$$

Calculate Z^1 for various present values P_i. Then, by entering the Z tables at Z_i^1 it is possible to find the probability that the expected present value E(P) is less than P_i.

3.2 ESTIMATING CASH FLOW ELEMENTS (X_{tj}).

In Section 2.2(i) a cash flow was defined as a random variable with finite mean μ_{tj} and variance σ^2_{tj}. Expressing the variable in this way infers that it has an underlying probability distribution which must be forecasted. This section focuses attention on the various forecasting techniques, the way probability distributions are derived and the types of distribution used by project appraisers.

3.2.1 Forecasting Techniques

Chambers et al. (1971) lists 18 basic forecasting techniques which may be grouped into three main categories:- qualitative techniques, time series analysis and projection, and causal models (see Table 2).

TABLE 2 Basic Forecasting Techniques

A. Qualitative Methods	B. Time Series Analysis and Projection	C. Causal Methods
1. Delphi Method	1. Moving Average	1. Regression Model
2. Market Research	2. Exponential Smoothing	2. Econometric Model
3. Panal Consensus	3. Box - Jenkins	3. Anticipation Surveys
4. Visionary Forecasts	4. X - 11	4. Input Output Model
	5. Trend Projections	5. Economic I/O Model
		6. Diffusion Index
		7. Leading indicator
		8. Life Cycle Analysis

(a) Qualitative Techniques. Qualitative techniques are used primarily when data are scarce. They use human judgment and rating schemes to turn qualitative information into quantitative estimates. The objective being to bring together in a logical, unbiased and systematic way all information and judgments which relate to the factors being estimated.

(b) Time Series Analysis and Projection. Time series analysis and projection are statistical techniques that are useful when several years' data are available and when relationships and trends are both clear and relatively stable. However, as the techniques are based solely on historical data they are useful only in the short term.

(c) Causal Models. Causal models use highly refined and specific information about relationships between system elements. The techniques are powerful enough to take special events into account and are typically revised continually as more knowledge about the system becomes available.

Often final decisions about uncertain future events are made after combining the information from several of the above techniques. As Chisholm and Whitaker (1971, p 2) point out:- "Forecasting the future remains in large measure an art. Like any artist, a good forecaster must be a skilled technician in whose hands the work of art evolves."

The forecasting techniques used most in project appraisal work fall into the qualitative type.

3.2.2 Deriving Subjective Probabilities

Whichever forecasting technique is chosen there is still the problem of describing the forecasted variable in an unambiguous and simple fashion. It is well recognised that subjective probability distributions fill this role. The distributions are termed subjective because of the uncertain nature of the future environment of which no man has positive knowledge. Savage (1962, p 11) defines subjective probability as the opinion of a person as reflected by his real or potential behaviour.

Use of subjective probabilities to describe uncertain future events has received its major stimulus through concern that traditional methods usually produce strongly biased estimates (Woods, 1966). One of the major aims of probability analysis is to reduce this bias.

Although there has been a gradual acceptance of the subjective probability distribution by decision makers the techniques used to derive them have been flexible and to many appeared to have some magical quality. However, Smith (1967) made an attempt to derive a formal model. His approach was rather complex but supposedly specific, logical, and consistent. Smith considered most previous attempts at deriving subjective probability distribution to be characterised as "hand-waving" techniques. He cites Hertz (1964) as giving perhaps the best general method for obtaining subjective probabilities but describes it as being not sufficiently definitive to comprise a consistently acceptable technique.

Essentially, Hertz used a series of meetings "to probe and question the experts" to ascertain the distribution. Smith's own method is based on a complex five step ranking procedure.

Morrison (1967) approached the problem in a different and simpler way. His method consisted of constructing a cumulative probability distribution through direct questioning of the expert. Woods (1966) also described a method essentially similar to that of Morrison.

Green (1967) suggested a simpler alternative to Smith. His method required the expert to distribute 100 points over the parameter interval shown on a diagram so that the resulting picture best reflected the uncertainty of the situation. However, Green considered the issue of finding subjective probabilities to be still wide open.

The increasing importance of Bayesian analysis (Schlaifer 1961) in decision making has prompted research into developing *a priori* distributions. Winkler (1967, 1968) has conducted several experiments to evaluate both mathematical and behavioural approaches to consensus decision making in the Bayesian context. He concludes that there can be no method that is more correct than any other.

One well documented method for deriving subjective probability distributions is the Delphi Method (Dalkey and Helmer 1963). It is widely used in corporate decision making in America, and has recently been adapted to technological forecasting (North and Pyke 1969).

Cole (1970) tested three methods for deriving subjective probability distributions. They were, firstly, the method suggested by Morrison as described earlier, and secondly that of Hillier (1963). Hillier's method relied on a normality assumption and required an estimate of the most likely value and the standard deviation. The third method suggested by Paris (1967) required the expert to estimate the most likely value, the maximum and minimum values, and the probability that the actual value would fall between the most likely and the maximum. Cole's results were inconclusive. He suggested that all three methods be used: First construct three subjective cumulative distributions then check these with the suppliers of data to see which is best. If none appeared correct then the procedure should be repeated until the experts are satisfied.

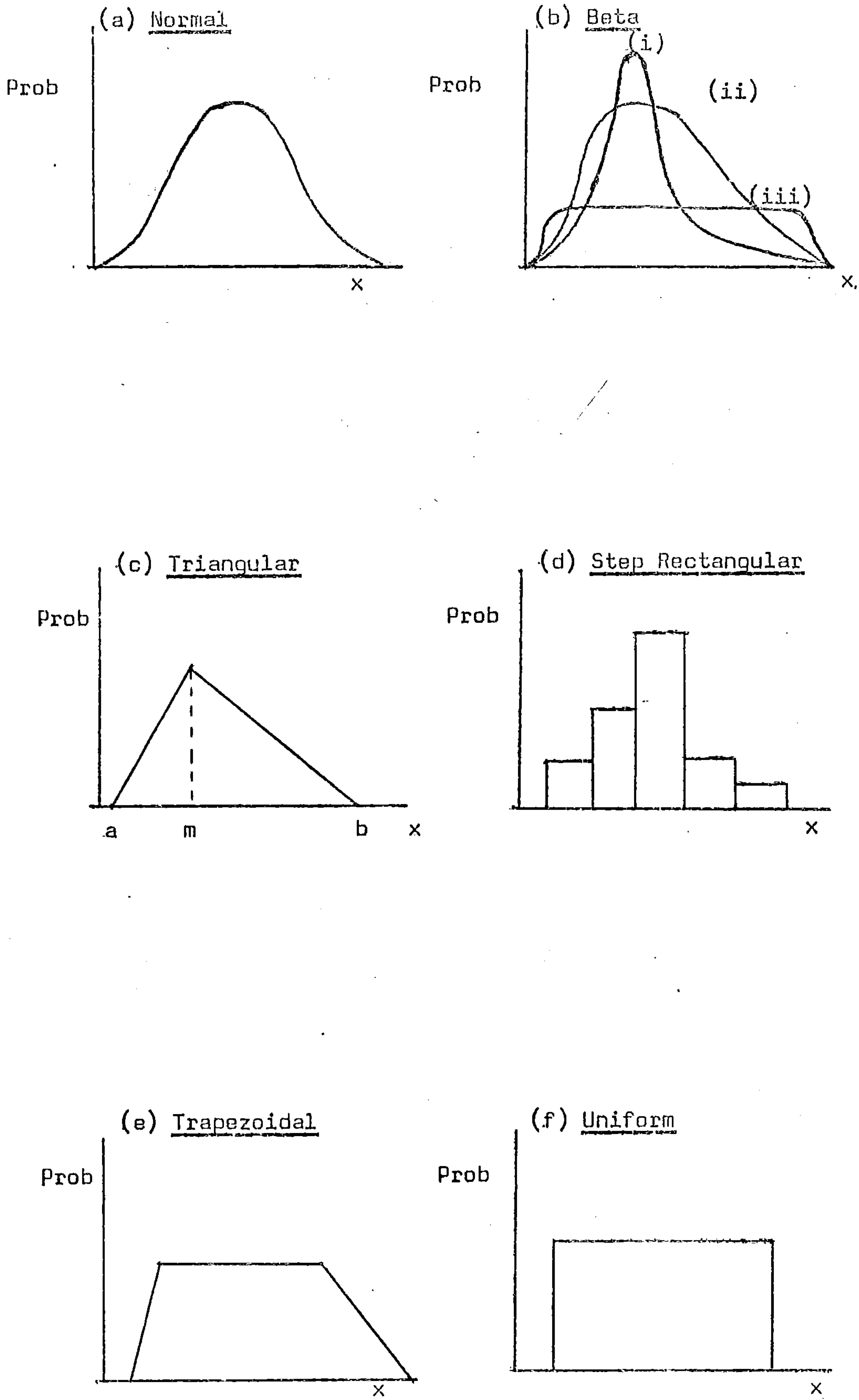
Pouliquen (1970) however, considers that deriving subjective probabilities can easily be done by the project appraisers themselves. Initially World Bank appraisal missions used two methods to arrive at subjective probability distributions. The first used was the so-called portrait approach (Pouliquen 1970 p 13), but this method has subsequently been dropped. The second approach, and their recommended procedure, leads to the step rectangular distribution. The procedure is iterative and leads to interaction between qualitative and quantitative judgment.

3.2.3 Choice of Distribution

It appears that the actual choice of a probability distribution is not as critical as has been thought by many people. Several writers, for example, Weibull (1951) has pointed out that it is not in fact possible to find the correct distribution anyway. As Pouliquen (1970) has stated: it is more a matter of finding a distribution which will make use of all the information available but not require more, than is, in fact available. Risk analysis does not attempt to find the true return, but only the appraisers' best estimate of it.

Sprow (1967) outlines several characteristics that would be desirable in a distribution. Firstly, it should be defined (uniquely if possible) by parameters which are unambiguous and easily understood by the estimators. Secondly, it should be capable of being skewed if desired, and thirdly, it should be amenable to mathematical analysis and computer manipulation. Sprow suggests that the triangular distribution (see Fig. 2 (c) comes closest to fulfilling these criteria. It is simple, uniquely defined by its range and mode and may be skewed.

Figure (2) Common Probability Distributions Assumed In Project Evaluation Studies



The formulae for the exact mean and variance of the triangular distribution are as follows (MacCrimmon and Ryavec, 1964 p 24).

$$\mu_x = 1/3 \ (a + m + b) \qquad \ldots\ldots\ldots \ (37)$$

$$\sigma_x^2 = 1/18 \left[(b-a)^2 + (m-a)\ (m-b) \right] \qquad \ldots\ldots \ (38)$$

where x = the value of the particular variable

a = the minimum value of x

m = the mode or most likely value of x

b = the maximum value of x

Also Cassidy, Rodgers and McCarthy (1970) show that the triangular distribution can easily be sampled by Monte Carlo computer techniques. The equations below give the value of a stochastic variable (x) as determined by random selection of the ordinate of the cumulative probability distribution F (x).

$$x = a + \left[F(x)\ (b-a)\ (m-a) \right]^{\frac{1}{2}} \qquad a \leqslant x \leqslant m \quad \ldots \ (39)$$

$$x = b - \left[\left(1 - F(x)\right)\ (b-a)\ (b-m) \right]^{\frac{1}{2}} \qquad m \leqslant x \leqslant b \quad \ldots \ (40)$$

Another distribution which has found widespread use in PERT Analysis (see section 2.4.5) is the PERT - Beta see Fig. 2 (b). It has several drawbacks when compared with the triangular distribution, nevertheless both Hertz (1964) and Hillier (1969) used it in their expository articles on Risk Analysis. The PERT - Beta is actually a special form of the true Beta distribution.

It is limited to taking a fat, flat shape, while the true Beta may take forms from quasi-uniform to quasi-delta. This is illustrated in Fig. 2 (b) where curve (i) is quasi-delta, curve (ii) PERT - beta and curve (iii) is quasi-uniform.

Because the PERT - beta is an approximation of the true Beta it is subject to errors in estimation. It is also less tractable to computer analysis when compared with the triangular distribution because it is not uniquely defined by its range and mode. Additional assumptions are required. Several authors have cast doubts on these assumptions, and on the statistics of the PERT - beta; for example, Grubb (1962) and MacCrimmon and Ryavec (1964). Moder and Rodgers (1968) have also found that the original 0-100 percentiles which define the range of the PERT - beta give biased estimates of the variance. They suggest that 5-95 percentiles limits be used when estimating the distribution subjectively. They recommend in this case that the variance be given by:

$$\sigma_x^2 = (b-a)^2 / 10.2 \qquad \dots\dots\dots\dots\dots (41)$$

World Bank staff (Pouliquen, 1970) reject both the triangular and beta distributions in favour of the step rectangular distribution. Pouliquen has tested the above three distributions along with the discrete, uniform, trapezoidal and normal distributions (see Fig. 2). In reviewing them Pouliquen points out that each distribution is geared to a particular situation and the choice should be made to make maximum use of available information.

The choice of distribution is not so important when the analytical technique is used. This is because only the first two moments (the mean and variance) of each distribution are taken into account when calculating the mean and variance of the present value. However, when simulation techniques are used, the third moment, which measures the skew of the distribution can be taken into account. Some writers, for example, Kryzanowski, Lusztig and Schwab (1972) also consider the fourth moment. This is a measure of kurtosis or the extent of the relative steepness of the ascent of the distribution in the neighbourhood of the mode. This is illustrated in Fig. 2 (b) where curve (i) and curve (ii) both have the same range and mode but are obviously quite different distributions.

As stated in section 2.5.1 most of the information about probability distributions is contained in the first two moments. Also, it becomes more difficult to subjectively estimate the higher moments. It is doubtful whether the forecaster would have enough information to estimate the fourth moment in most cases, although Monte Carlo simulation techniques are able to incorporate this extra information into the final distribution of the present value.

We now turn to the problem of correlation.

3.3 CORRELATION

Where some dependency is known to exist between random variables then the variables can be conceived as being statistically correlated. Correlation occurs whenever some underlying factor or factors cause the variables to move together in a systematic way. It is hard to detect and evaluate but if not taken into account may lead to a completely wrong interpretation in the analysis.

As indicated in section 2.2 there are two types of correlation - auto-correlation and cross-correlation. In this section several methods which can be used to evaluate these relationships are outlined.

3.3.1 Auto-correlation

Auto-correlation can be thought of as the relationship that occurs between similar cash flow elements from one period to the next. To determine the variance of auto-correlated cash flow streams the auto-correlation coefficients between periods must be specified. The method is described below (Bussey and Stevens 1972).

(a) Formal Approach. Let X_{tj} and $X_{t'j}$ be correlated cash flows in periods t and t'. Then the covariance between X_{tj} and $X_{t'j}$ is defined by the relationship -

$$\text{Cov}\ (X_{tj}\ X_{t'j}) = \rho_{tt'}\ \sigma_{tj}\ \sigma_{t'j} \qquad \ldots\ldots (42)$$

where $-1 \leqslant \rho_{tt'} \leqslant +1$ is the simple correlation coefficient and σ_{tj} and $\sigma_{t'j}$ are the standard deviations in periods t and t' successively.

The variance of the cash flows is given by:-

$$V(P) = \sum_{t=0}^{n} \frac{\sigma_t^2}{(1+i)^{2t}} + 2 \sum_{t \neq t'} \frac{\rho_{tt'} \sigma_t \sigma_{t'}}{(1+i)^{t+t'}} \quad \ldots (43)$$

In the independent case all $\rho_{tt'} = 0$ and thus the second summed term drops out.

Use of equation (43) requires that $\rho_{tt'}$ must be specified. To determine the auto-correlation coefficient $\rho_{tt'}$ it is necessary to find the causative or dependency relationships between the two periods for the particular flow. For example assume that gross revenue G_t is auto-correlated, i.e. G_t is correlated with G_{t-1}, $G_{t-2} \ldots G$ and with $G_{t+1} \ldots G_n$. Different mathematical relationships can be derived to explain this correlation (Box and Jenkins, 1970).

One of the simplest is the first order Markov process. This merely states that whatever influences G_t in period t originates solely from period t-1 etc. To use this model all that is required is an estimate of the one-period lag coefficient ϕ_1 between G_t and G_{t-1} then all other auto-correlation coefficients can be calculated from the following relationship.[15]

$$\rho_{tt'} = \phi_1^{|t-t'|} \quad \ldots\ldots\ldots\ldots (44)$$

The one-period lag coefficient is given by

$$\phi_1 = \frac{\sum_{t=2}^{n} (G_t - \hat{G}_t)\ (G_{t-1}) - \hat{G}_{t-1})}{\sum_{t=2}^{n} (G_{t-1} - \hat{G}_{t-1})^2} \quad \ldots\ldots (45)$$

---

[15] McArthur (1971) derives this by using the chain rule in the theory of path coefficients.

where $\hat{G}_t$ is the estimate of revenue in year t (obtained from a prior linear regression) and G_t is the observed revenue in year t (analyst's subjective estimates). This is a very simple model, for more sophisticated ones see Box and Jenkins (1960).

The above discussion refers to only one variable being auto-correlated, but in a more realistic analysis other variables are likely to be also. As the number of auto-correlated variables increases the auto-correlation coefficient matrix becomes very complex. In order to simplify the mathematics Bussey and Stevens have developed a hurdling procedure.

(b) Bussey and Stevens' Hurdling Procedure. The authors state that where the analysis does not justify detailed research into the underlying phenomena (e.g. correlation among sales, variable costs, fixed costs, etc.) or detailed information on the cash flow is not available the simplified method below could be justified. Their procedure is to estimate the auto-correlation coefficient assuming that the relationship between the variables is no more complex than first order Markov, and that subjective estimates of the optimistic, most likely and pessimistic cash flow increments (denoted Y_0, Y_m and Y_p respectively) for each period are available.

The method relies on a normality assumption. It assumes that the subjective probability distribution chosen is close enough to the normal distribution to be more reliable than choosing more subjective estimates (see section 3.3.1 (c)).

The maximum likelihood estimation principle is used: it makes maximum use of the information within the data (pessimistic, most likely, optimistic). The assumption is made that the Y_p, Y_m and the Y_o for each period are samples from *a priori* multivariate normal populations.

$$\text{i.e.} \quad \left[Y_p(t),\ Y_m(t)\ Y_o(t) \right] \propto MN\left(\mu_{y(t)},\ \sigma^2_{y(t)}\right) \quad \ldots (46)$$

It is also assumed that the samples are extracted successively for $t=0,1,2,\ldots,n$. and that they are correlated samples, with the time co-relationship being first order Markovian. Under these assumptions the appropriate maximum likelihood estimator of the population lag coefficient is given by:

$$\hat{\phi} = \frac{\frac{1}{N-1} \sum_{t=1}^{n} W_t W_{t-1}}{\frac{1}{N-2} \sum_{t=1}^{N-1} W_t^2} \quad \ldots\ldots\ldots (47)$$

where W_t = the first differences in the sampled cash flow stream; i.e. $Y_p(t) - Y_p(t-1)$, $Y_m(t) - Y_m(t-1)$, $Y_o(t) - Y_o(t-1)$ indexed at time t. W_{t-1} = the first difference (as above) indexed at time $t-1$. N = the number of data sample points, which equals $3(n-1)$ when all three estimates of cash flow are used. Once ϕ has been estimated the $\rho_{tt'}$ are given by equation (44). Then the variance can be calculated from equations (24) or (30).

(c) Hillier's Approach. (also Canada (1971) p 184)
Hillier (1969) has a slightly different approach to Bussey and Stevens, it uses a normality assumption but requires the use of additional subjective judgments. The approach assumes a joint bivariate normal distribution (Mood and Graybill 1963 p 202). Given two random variables X_1 and X_2 then

$$E\,(X_2 \mid X_1 = X) = E(X_2) + \frac{\rho\,\sigma_2}{\sigma_1}\left[X - E(X_1)\right] \quad . \; . \quad (48)$$

after normalising this becomes

$$\frac{E(X_2 \mid X_1 = X) - E(X_2)}{\sigma_2} = \rho\left[\frac{X - E(X_1)}{\sigma_1}\right] \quad . \; . \; . \; . \quad (49)$$

Equation (49) states that the expected value of X_2 given X_1 is just ρ times the value of X_1 and it provides the basis for deriving subjective estimates for ρ.

Given the X_1 lies k standard deviations above its mean then the expected value of X_2 will lie ρ k standard deviations above its unconditional mean.

Where the distributions are not normal but skewed, as could be the case with the triangular or Beta distributions, the above will not generally hold. However, according to the principle of least squares equation (49) is still the best estimate of ρ.

Using equation (49) values can be inserted from the subjective probability distributions to obtain estimates of ρ. Hillier suggests that the most meaningful values of X_1 are the bounds (optimistic, pessimistic). Thus, they should be used to select the conditional value of X_2 to derive a value for ρ. Hillier further suggests that an unweighted average of the two values of ρ should be used as the final composite estimate.

This method can be used for the estimation of both auto-correlation and cross-correlation coefficients.

(d) Tests for Significance. The lag coefficients ($\hat{\phi}$) can be tested for significance by using Normal Tables. One test would be to see whether $\hat{\phi}$ is significantly different from zero at the 95 percent confidence level (i.e. Z = 1.96). Box and Jenkins (1970, p 281) show that the variance of the lag coefficient is approximately

$$\mathrm{Var}\ (\hat{\phi}) \approx \frac{1}{n}\ (1 - \phi^2) \qquad \ldots\ldots\ldots\ldots \quad (50)$$

Therefore the standard error is

$$\mathrm{S.E.} \approx \sqrt{\frac{1}{n}\ (1 - \phi^2)} \qquad \ldots\ldots\ldots\ldots \quad (51)$$

$$\therefore\ Z^1 = \frac{\phi - 0}{\mathrm{S.E.}} \qquad \ldots\ldots\ldots\ldots \quad (52)$$

If Z^1 is greater than Z (Z = 1.96) then it would be accepted that the lag coefficient is different from zero and that auto-correlation exists and should be taken into account.

3.3.2 Cross-correlation

At times when general economic factors affect the activities of a project in the same period then the net cash flow of these activities will be cross-correlated. That is, when dependency exists between two cash flows X_{tj} and X_{ti} in the same period then the covariance between them must be specified. This was given in equation (19) section 3.1.2 (a). The covariance can be stated as :-

$$\mathrm{Cov}\,(X_{ti}\,,\,X_{tj}) = \rho_{ij}\;\sigma_i\;\sigma_j \quad \ldots\ldots (53)$$

where $-1 \leqslant \rho_{ij} \leqslant +1$. If $\rho_{ij} = 0$ for all variables then the covariance term drops out, otherwise the problem is to estimate the cross-correlation coefficient ρ_{ij}.

(a) Estimation by Regression. Bussey and Stevens set out assumptions and methodology which make this a practical approach. They make three major assumptions.

Firstly, the correlation between two cash flow increments X_{ti} and X_{tj} $(i \neq j)$ in the same period is the only relationship between the two variables. (This is the usual practical situation when underlying economic factors tend to push activities either up or down together).

Secondly the partial correlation coefficient $\rho_{ij}(t)$ is constant over all time periods t=0, 1, 2, 3,..., n. (This allows the regression of one activity X_{ti}, against another, X_{tj}, for t=0, 1, 2, 3,..., n to obtain an estimate of the correlation coefficient relating the two activities.

The regression equation is

$$\hat{X}_{tj} = E(X_{tj}) + \hat{\beta}\left[X_{ti} - E(X_{ti})\right] \quad \ldots\ldots\ldots \quad (54)$$

where $\hat{X}_{tj}$ is the estimate of X_{tj} and $\hat{\beta}$ is the slope of the linear regression function.

The third assumption is that the two cash flows be considered normally distributed. (If X_{ti} or X_{tj} are estimated using triangular or Beta functions then the model would not normally apply. However, equation (54) still provides the best linear estimate of X_{tj} via the method of least squares).

If equation (54) is acceptable then the Pearsonian sample correlation coefficient r_{ij} given by:-

$$r_{ij} = \hat{\beta}\left[\frac{S_{ti}}{S_{tj}}\right] = \frac{\sum (X_{tj} - Z)\ \ (X_{ti} - Z)}{\left[\sum (X_{tj} - Z)^2\ (X_{ti} - Z)^2\right]^{\frac{1}{2}}} \quad \ldots \quad (55)$$

where S_{tj} and S_{ti} are the sample standard deviations, r_{ij} is the best linear estimator of the correlation coefficient and $Z = (\bar{X}_i + \bar{X}_j)/2$ then $\hat{\rho}_{ij} = r_{ij}$

(b) <u>Tests for Significance</u>. The estimate of the cross correlation coefficient $\hat{\rho}_{ij}$ may be tested for significance by using the T test. One test would be to see whether $\hat{\rho}_{ij}$ is significantly different from zero at the 95 percent confidence level. The test performed (Johnston, 1963, p 33) is

$$t = \frac{\rho\sqrt{n - 2}}{\sqrt{1 - \rho^2}} \quad \ldots\ldots\ldots\ldots \quad (56)$$

If the $|t|$ is greater than $t_{.05}$ with $(n-2)$ degrees of freedom then it may be inferred that there is a significant correlation between X_i and X_j and therefore the covariance between them should be taken into account.

(c) Subjective Estimates. Where the past cannot be taken as a reliable guide to the future, as is assumed above, then a subjective estimation procedure as outlined in section 3.3.1 (c) may be the only solution.

3.4 SUMMARY

In this chapter the analytical method for handling risk and uncertainty has been developed through the use of probability calculus. We have seen how the techniques have evolved from being able to calculate the variance of simple summed random variables to handling complex combinations of products of random variables. Recourse to the Central Limit Theorem has allowed probability statements to be made about the overall distribution of the combined variables.

Establishing the actual probability distribution of the random variables has attracted much attention in the business world. Forecasting techniques range from intelligent guessing to the use of complex econometric models, but as yet there seems to be no universally agreed method for deriving the subjective probability distributions of the variables. One point does seem clear however. This is that the resultant shape of the distribution is not nearly as important as the way in which the variables relate to each other within and between periods. It is this aspect of inter-dependencies between variables that is most important to establishing useful estimates of the decision variables.

The analytical technique incorporating Taylor's approximation does offer considerable advances over the traditional methods. The earlier criticisms advanced in section 2.4.4 have been met. Judgment is now applied to the underlying assumptions in the project rather than to the results of the analysis. The variability of the project is measured by a single overall indicator (the variance), not by a number of criteria. The technique allows for inter-action between the variables which make up the project. A quantitative assessment of risk is made rather than qualitative statements and lastly the basic framework is laid for consistent analysis project to project and analyst to analyst.

There are, however, two further important aspects of risk and uncertainty which cannot be handled by analytical means. The next chapter on simulation looks into these aspects along with the methodology and problems of technique.

CHAPTER IV

SIMULATION

4.1 INTRODUCTION

It has been shown that the analytical techniques outlined in chapter three can overcome most of the deficiencies of traditional methods for handling risk and uncertainty. There are however, two further refinements that cannot be handled by this technique. These are as follows:

(i) When skew and perhaps higher moments of the probability distributions deviate substantially from the normal, the overall distribution of the present value may also be non-normal. Also combining the products of random variables may cause abnormality. Where this affects projects differently then choices on the basis of normality could give incorrect decisions.

(ii) Capital investments usually involve dynamic decisions over time. For example, high revenues in year t-1 could well be reflected in higher on-farm investment in year t which would appear as increased revenue in year t +1 etc.

Both of these aspects can be handled by using simulation techniques incorporating Monte Carlo Methods. Naylor (1971) defines simulation as a numerical technique for conducting experiments with certain types of mathematical models which describe the behaviour of a complex system on a digital computer over extended periods of time. Simulation has become increasingly popular among economists and management scientists as a tool for analysing complex systems and has been applied to a number of areas in agricultural research (Dent and Anderson, 1971). Authoritative reviews on the relevance, methodology, application and prospects for the use of system simulation in agriculture may be found in Charlton and Thompson (1970) and Anderson (1974).

4.2 SIMULATION IN PERSPECTIVE

System simulation is hardly a practical operational procedure especially when compared with other mathematical techniques such as linear programming. Simulation modelling has been restricted almost exclusively to academics and researchers. The methodology is far from being settled and there are many difficulties to be overcome. The most real of these being the difficulties in identifying and quantifying relationships, in defining the bounds of the system to be studied, in collecting data in the correct form and in validating, analysing and interpreting the results. The justification for proceeding with simulation in this thesis is that by introducing the stochastic and dynamic aspects outlined above it is possible to represent the real world in a more telling manner. Whether the extra time and expense involved in obtaining this extra information is worthwhile will be discussed later.

The evaluation of capital projects in agriculture by systems analysis has not received much attention, but the potential use of system simulation for this purpose could be great for very large projects. Complex models of specific agricultural systems have been developed which could be applied to the capital budgeting problem. For example, Flinn (1971) has developed a model of a crop-irrigation system designed to integrate the economic aspects of both the demand and supply of irrigation water with the physical and institutional constraints found in water resource systems. His model consists of three major components as follows:

(i) those factors determining the level of atmospheric demand for moisture;

(ii) those concerned with the availability of moisture for the crop; and

(iii) the interaction between the supply of, and demand for water on economic yield.

The model involves dependencies between the components and time dependent causal relationships between the prior environment and the current growth state of the crops. This is a very sophisticated model of which Flinn states - possibly one of the most exciting uses may be in evaluating the expected agricultural benefits from a proposed irrigation development.

However, it is my view that this type of model is so far removed from presently used techniques that it would not be practical to institute it as a technique to evaluate capital projects in the short term. Even in the light of the current problem of allocating water for irrigation on the relatively large Canterbury Plains Schemes, to which Flinn's type of model would be well suited, the difficulties of implementation would be too great.

Rather, it is suggested that a much higher level of aggregation of a project would be appropriate. Flinn gets down to the detail of simulation such components as soil moisture tension, transpiration, and moisture stress severity on the plant to obtain the quality and quantity of crop yields from the system. Here it is proposed that the major components of the model would be the financial budgets which represent the various possible activities in a project, such as sheep, cattle, wheat, white clover etc.

Thus we have withdrawn from the position of carrying out a full systems analysis involving many components to simulation a very simple system with few components using Monte Carlo methods.

4.3 MONTE CARLO METHODS

Monte Carlo Methods are used to handle those variables which are subject to uncertainty. The method is discussed fully in Hamersley and Handscomb (1964). In essence it consists of specifying the class of probability distribution for each stochastic variable together with its parameters. Sample values of the variable are then drawn from the distribution. The technique uses random sampling where numbers uniformly distributed from 0.0 to 1.0 are assigned to the alternative values of the stochastic variable in accordance with the specified probability distribution. Random sampling is usually carried out in the computer by system's supplied sub-programs which generate pseudo-random numbers. A number of different techniques can be used to generate pseudo-random numbers and these are discussed fully in Ralston and Wilf (1967).

4.4 THE STRUCTURE AND PROPERTIES OF A SIMULATED AGRICULTURAL PROJECT

The Monte Carlo Simulation technique is a way of representing a proposed agricultural project as a mathematical model on a computer. The model would have the following types of properties and structure.

It is made up of a number of interrelated time dependent components which represent the various possible activities in the project. The behaviour of the components within the system are described by the functional relationships. These are represented within the computer program as arithmetic statements, function subprograms and subroutines.

Inputs into the components may be stochastic, non-stochastic or policy variables or parameters. For example, in the sheep component, the input wool price would be independent, exogenous and stochastic, as it is generated outside the system and is subject to a large degree of uncertainty. On the other hand, the planning horizon which may be known with more certainty would be termed an independent, non-stochastic input variable. The policy of running N sheep, that is the flock size, would be a policy variable and thus be subject to experimentation by the analyst. Parameters are values which are constant for one simulation. They are required for the functional relationships and for the generation of stochastic variables. For instance, the process of sampling from a probability distribution of wool price may require such parameters as its mean and standard deviation.

Variables that are derived within the system are called endogenous. Thus, wool income would be an endogenous dependent variable and if a function of input stochastic variables it too would be stochastic.

The sheep component has an output or response variable of revenue from sheep. This is simply a function of wool weight times price, plus lambs sold times value, less variable costs, all times the number of sheep. Thus, the frequency outcome of sheep revenue is derived from the functional relationship between the input and output variables of the sheep component.

Once the output has been generated for all the components in a particular year they are combined to give the net cash flow for that year. The procedure is then repeated until the cash flows have been generated for the full project life. The net cash flow is then used to calculate the decision variables (present value or internal rate of return) for that particular run. These values are stored and the model run another several hundred times to establish the overall distribution of the decision variables.

Although the model as described above may appear to be relatively straight forward to implement there are several major problems associated with it. The most important of these are stochastic specification, dependency relationships, dynamic consideration, and verifying and validating the model. In the next section these problems will be discussed, but first it is shown how the basic Monte Carlo Model has been developed and applied to capital budgeting decisions.

4.5 THE APPLICATION OF SIMULATION TO CAPITAL INVESTMENTS

The stimulus to use simulation to evaluate uncertain capital investments came about through the failure of simpler techniques to give satisfactory results. By using Monte Carlo Simulation it is possible to extract the maximum amount of information from the available data and forecasts of the future. The aim being to give as clear a picture of the relative risk and the likelihood of obtaining a positive present value in the light of the uncertain future. The basic technique is relatively simple but requires access to a digital computer to be practicable. The basic model will now be outlined and then the methods used to overcome the problems of dependencies, subjective probabilities, dynamic considerations and model checking will be discussed.

4.5.1 The Basic Model

One of the earliest attempts at Monte Carlo Simulation (Hess and Quigley, 1963) was not computer orientated. The model is based on sampling from the normal distribution where the value of the sampled variable (V) is given by:

$$V = \mu_V + \sigma_V \text{ (RN)} \quad \ldots\ldots\ldots \text{(57)}$$

where μ_V equals the mean value of the variable, σ_V equals its standard deviation and RN is a random normal deviate drawn from the random normal tables.

It was Hertz (1964, 1968) however, who gave the greatest impetus to the acceptance of risk analysis. His method consists of the following three steps.

(a) Estimate the range of values for each of the uncertain variables and the likelihood of occurrence of each value within the range. That is, derive the subjective probability distribution for each variable.

(b) Select at random one particular value at each of the variables. Then combine the selected values of the variables taking into account any dependencies. This will give the present value (or IRR) of that particular run.

(c) Carry out step (b) over and over again to define and evaluate the odds of occurrence of each possible present value or IRR. When completed this procedure gives the distribution of the present value or IRR and therefore the expected present value and the variability of the present value.

This three-step procedure can be carried out to evaluate all the alternative investments open to the decision maker. It is then possible to evaluate different combinations of investments to see how they perform in meeting the objectives of the decision maker.

Hertz emphasises that the less certainty there is about an estimate the more important it is to consider its possible variation. The amount of knowledge, or lack of it, must play an important role in deciding between different investments. The simulation approach is designed to take account of the type of information which has in previous methods been ignored, mainly because of its highly uncertain nature.

Kryzanowski, Lusztig and Schwab (1972) outline a computer algorithm which is based on the Hertz model. The algorithm sets out clearly how to implement the procedure. Figure 3 represents a slightly modified version[16] of it presented in flow chart form.

4.5.2 Probability Distributions

One major advantage of Monte Carlo Simulation is that it allows full use of all the information obtained in the probability distribution of the simulated variable. For example Kryzanowski, Lusztig and Schwab (1972) outline a procedure to derive what could be called an empirical distribution. It is estimated subjectively and specifically takes into account the first four moments of the distribution: the mean, standard deviation, skew and kurtosis. Their model also allows for growth to be a stochastic variable itself, as well as for the growth of the stochastic variable over time. The effect of positive growth on a stochastic variable can be seen in Fig. 4. It is their contention that over time the entire distribution shifts to the right and its dispersion increases.

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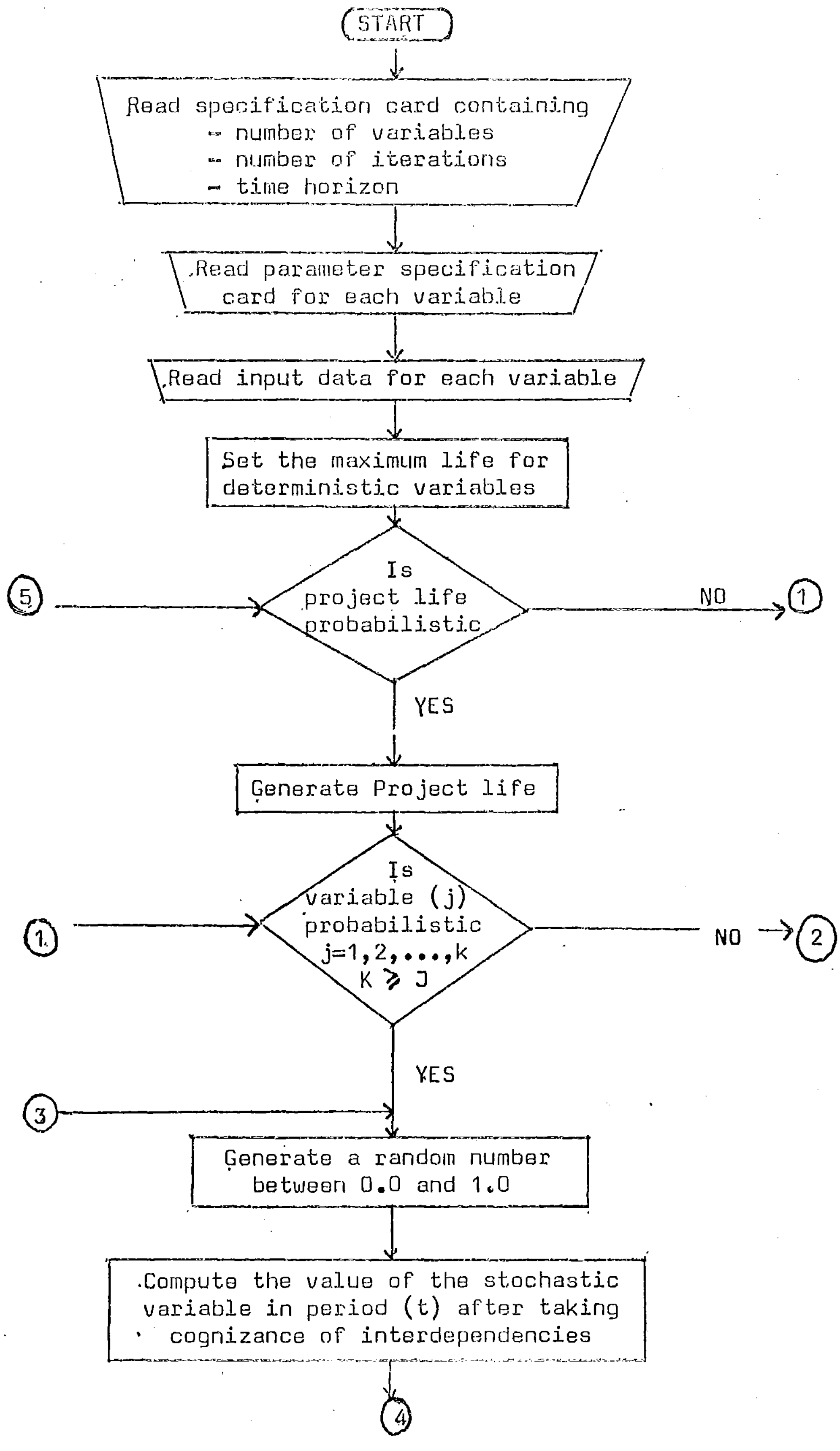
[16] As this thesis is concerned with project evaluation from the National point of view the original tax considerations have been omitted.

Fig. 3 A MONTE CARLO COMPUTER ALGORITHM



contd

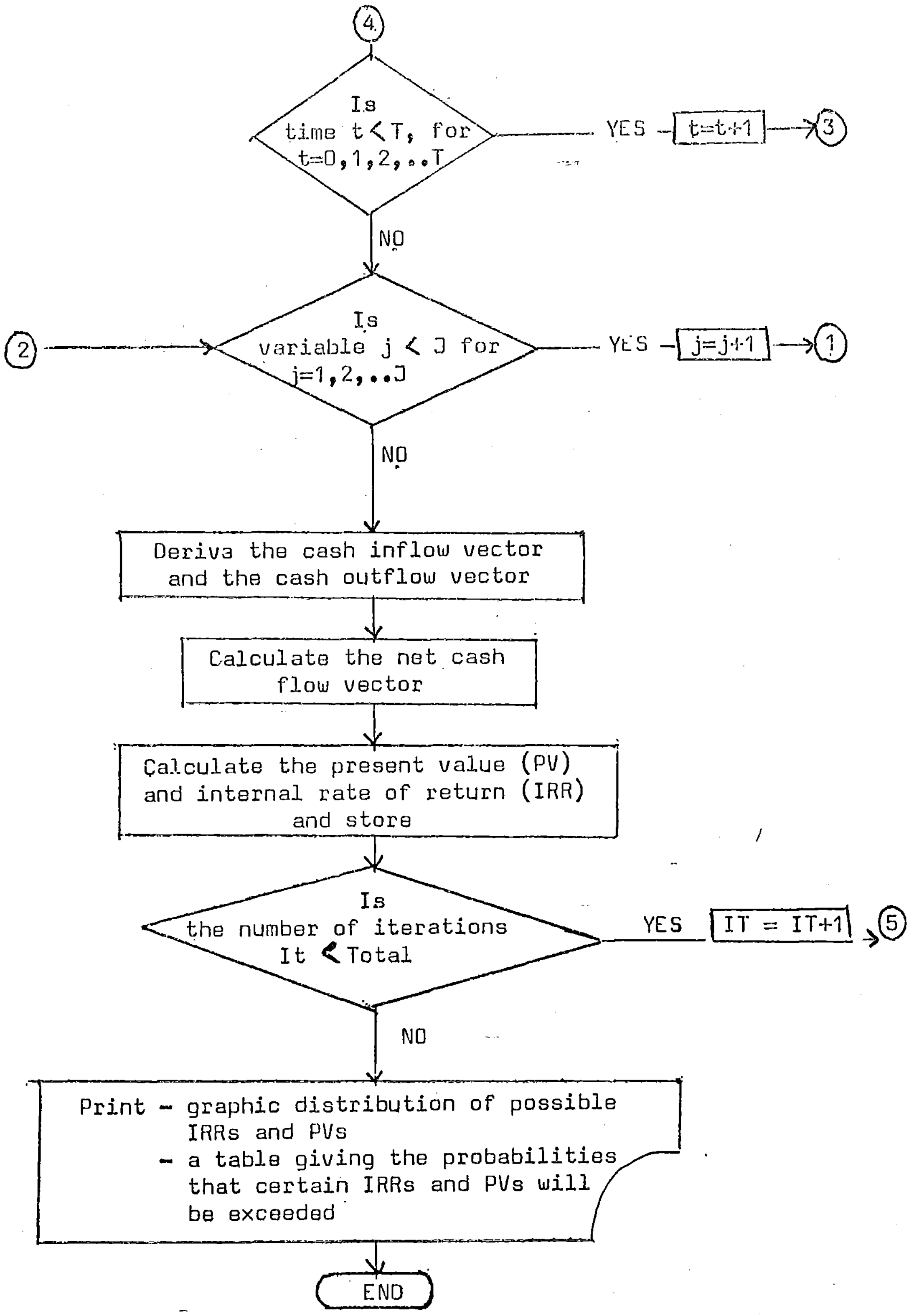
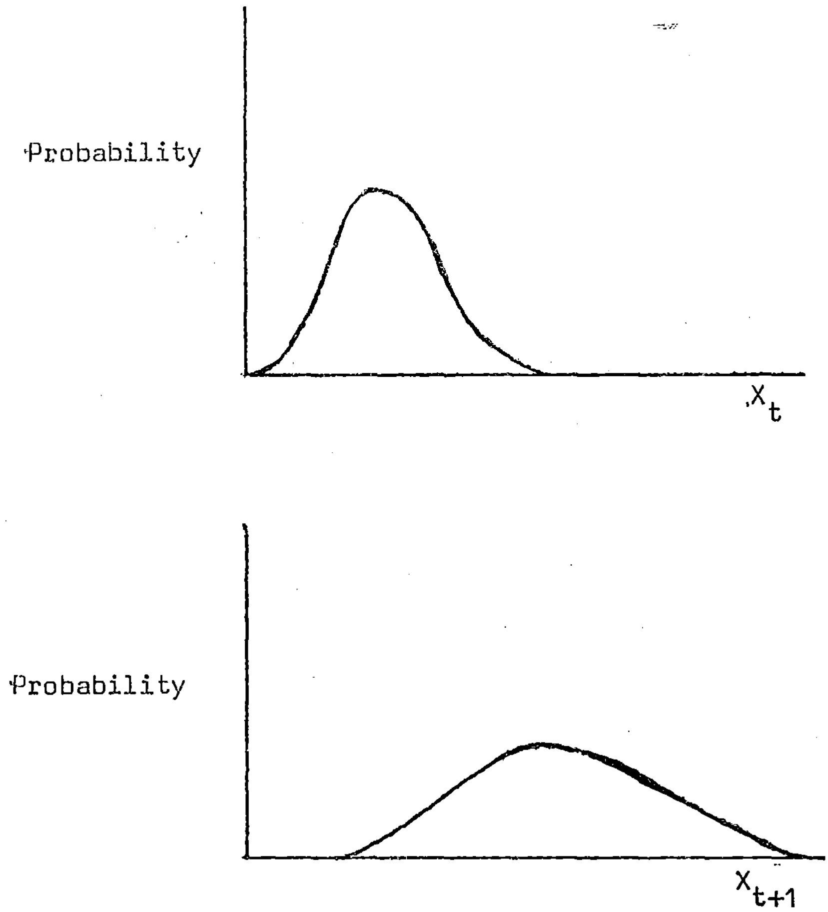

4
Is time t < T, for t=0,1,2,..T
YES
t=t+1
3
NO
2
Is variable j < J for j=1,2,..J
YES
j=j+1
1
NO
Deriva the cash inflow vector and the cash outflow vector
Calculate the net cash flow vector
Calculate the present value (PV) and internal rate of return (IRR) and store
Is the number of iterations It < Total
YES
IT = IT+1
5
NO
Print - graphic distribution of possible IRRs and PVs
- a table giving the probabilities that certain IRRs and PVs will be exceeded
END

Figure 4 Time Growth in a Probability Distribution



As was outlined earlier in section 3.2.3 many theoretical distributions have been used in simulation. The actual choice of distribution must lie finally with the amount of information available to specify it.

4.5.3 A Special Case - The Multivariate Normal Distribution

One of the most widely used probability distributions is the normal distribution. Along with the lognormal it shares a major advantage over other types of distributions for simulation. This is that it may be simulated directly from standard normal numbers once the means, standard deviations and correlation coefficients have been given. The simulation of other distributions is more difficult because standard numbers are not available. Whereas random normal variates can be simulated by a simple transformation (see equation 57) the simulation of non-normal distributions requires more complex procedures such as outlined by Kryzanowski, Luszitig and Schwab (ibid.).

Also simulations from the multivariate normal distributions (Naylor, 1966, pp 97-99) can be used as a check against Taylor's approximation for estimating the variance of the present value. The procedure is now outlined.

The multivariate normal distribution is defined for a vector of random variables X where each component of the vector is a random normal variable x_i with given mean μ_i and variance σ_{ii}. Assuming the components of the vector are correlated then the variance - covariance matrix V, is required to generate the random normal vector.

V is given by :-

$$V = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1m} \\ \vdots & \ddots & \vdots \\ \sigma_{m1} & \cdots & \sigma_{mm} \end{bmatrix} \qquad \ldots\ldots\ldots (58)$$

where σ_{ii} denotes the variance of the i^{th} component and σ_{ij} denotes the covariance of the i^{th} and j^{th} components.

Generation of the random normal vector utilizes a theorem which states that if Z is a standard normal vector (with mean zero and standard deviation one) there exists a unique lower triangle matrix C such that

$$X = Cz + \mu \qquad \ldots\ldots\ldots (59)$$

That is

$$X \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = C \begin{bmatrix} \cdot & & & \\ \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} Z \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} + \mu \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad \ldots (60)$$

The lower triangular matrix C is obtained from V by using the square root method. This method relies on the relationship of -

$$V = C \, C' \qquad \ldots\ldots\ldots (61)$$

That is

$$V \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} = C \begin{bmatrix} \cdot & & & \\ \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} C' \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & & \cdot \end{bmatrix} \qquad \ldots . (62)$$

Equation (59) generates all the multivariate normal variables within a period, subject to the within period dependency relationships specified by the cross correlation coefficients. To generate auto-correlated variables, as well, an addition relationship is required. This may be stated as follows:

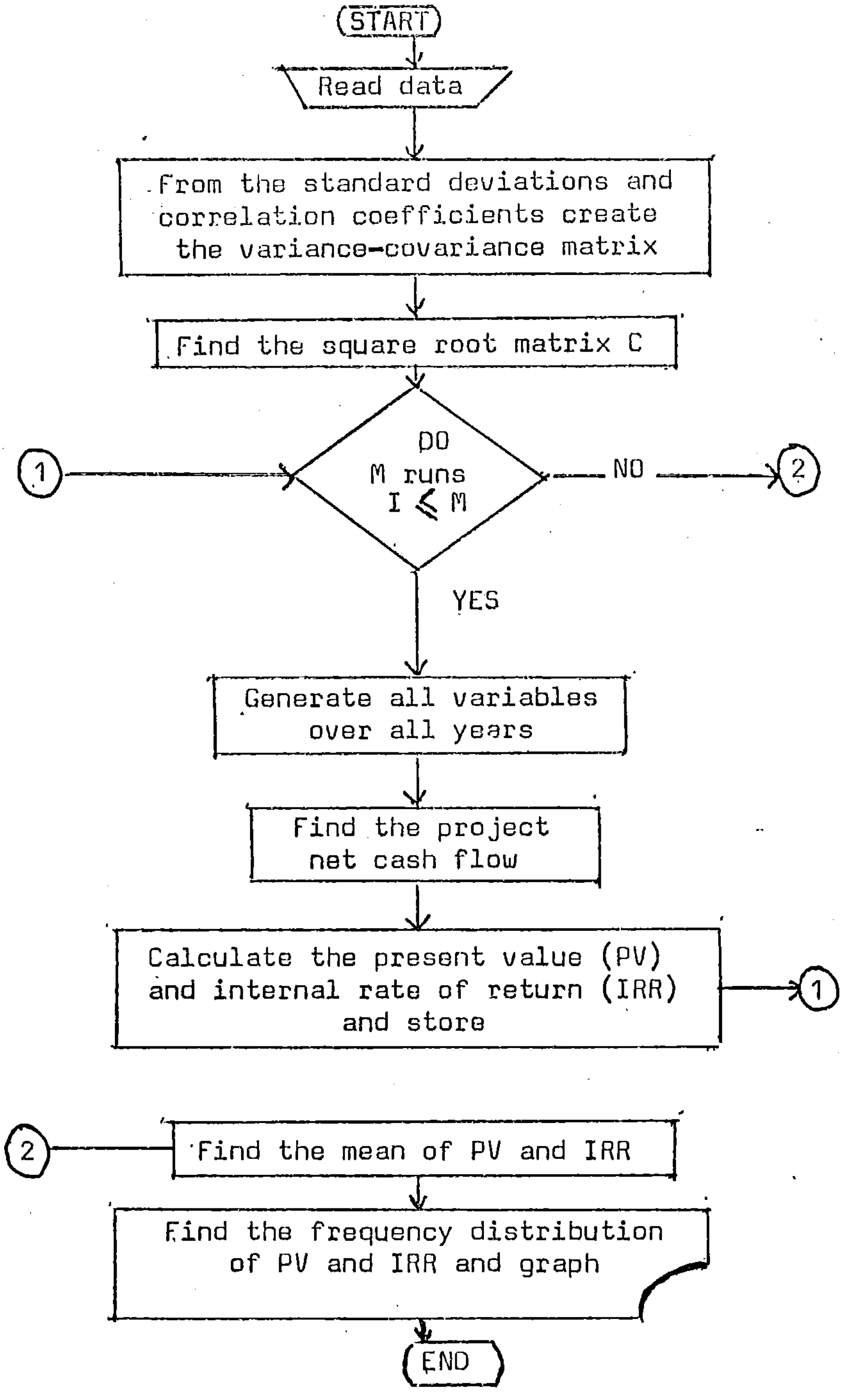
$$\hat{x}_{tj} = r_j \; (x_{t-1j} - \mu_j) \; + \mu_j \qquad \ldots\ldots . (63)$$

where $\hat{x}_{tj}$ is the auto-correlated variable j in period t, r_j is the auto-correlation coefficient, μ_j is the mean of the variable and x_{t-1j} is the random normal variable in period t-1. Thus within period t auto-correlated variables are generated subject to the value of the variable in period t-1. Then equation (59) is used to generate the random normal variables within that period.

The logic of the computer programme that generates random normal variables for risk analysis is shown in flow chart form (see Fig. 5).

Fig. 5 Monte Carlo Simulation Using The Multivariate Normal Distribution

Flow Chart



4.5.4 Dependencies

The problem of dependencies (or correlation) is tied to the level of aggregation of the cash flows. The level of aggregation refers to the detail encompassed in the analysis. For example, in the benefits to agriculture there could be returns for sheep, cattle and crops. The returns to sheep can further be subdivided into returns from wool and meat. Then the returns to wool may be further subdivided into price and quantity. The problem is to decide at what level to stop subdividing. Clearly there is a practical limit. The advantages of disaggregation are that when the components are broken down it is easier to specify the parts and put estimates on them, then build back up to a distribution of the whole. The disadvantage is that the correlation or interrelationships between variables and over time become more difficult to specify. There is a trade-off between the advantage of clarity of judgment and of avoiding the problems of disaggregation. As Pouliquen (1970) asserts - correlation is more important than the shape of the distribution. Thus he opts for a rather high level of aggregation. On the other hand the major advantage of risk analysis is that it permits disaggregation.

Specifying correlations quantitatively is usually found to be quite hard to do on a subjective basis. Thus, Pouliquen suggests that where the underlying relationships are hard to specify the analysis should be run at optimistic and pessimistic levels of correlation. This will test the sensitivity of the result to changes in these coefficients.

Kryzanowski *et al.* (1972) take a more rigorous view by sampling for correlation on a subjective basis. That is, rather than asking the forecaster for specific values of the correlation coefficients, ask for ranges wherein they are likely to lie. These can then be simulated. The authors found it necessary to be quite specific when identifying dependencies and the degree of intensity of association. Their computer program will incorporate or ignore dependencies during a particular period. This is done through *keying*. One variable is keyed to another - called the base variable. Two types of dependency are distinguished : direct and indirect. When direct, if the base variable is low then the keyed variable will be low. When indirect, if the base variable is low then the keyed variable will be high.

Also three levels of intensity of association are allowed. These are slight, moderate and high. Where the random number (which becomes the cumulative probability for the variable) generated for the base variable is $\leqslant 0.50$ then if the intensity is :-

(a) slight - then the probability that the cumulative probability of the keyed variable is $\leqslant 0.50$ is 0.60 for direct and 0.40 for indirect dependency.

(b) moderate - then the probability that the cumulative probability of the keyed variable is $\leqslant 0.50$ is 0.75 for the direct and 0.25 for the indirect dependency.

(c) high - then the probability that the cumulative probability of the keyed variable is $\leqslant 0.50$ is 0.90 for direct and 0.10 for indirect dependency.

The procedure works correspondingly if the base variable is $>$ 0.50. The above procedure represents a considerable degree of sophistication and requires a great deal of emphasis during the system analysis phase of the simulation.

When simulation is used to evaluate uncertain investments it is critical that special care be taken to specify dependencies if probability statements are to be made about the resultant present values. Failure to do this will result in a meaningless probability statement.

4.5.5 Dynamic Considerations

Investment over time is dynamic in nature. The system is subject to non-controllable stochastic influences such as market prices, feed supply and animal productivity. When allowances are made in the model for these dynamic considerations then use of analytical techniques and recourse to the Central Limit Theorem is no longer possible. An example of this is given by Cassidy, Rodgers and McCarthy (1970). In their example the authors have allowed the possibility of "poor strikes" in the case of pasture establishment, thus retarding project output. They state that this constitutes divergence of paths and negates use of the Central Limit Theorem.

Although analytical techniques are ruled out when dynamics are introduced Monte Carlo Simulation is able to incorporate it and offers a practical solution to the problem.

4.5.6 Checking the Model

Once the computer program has been written the model must be checked. In systems simulation this is a major problem and involves many practical, theoretical, statistical and philosophical complexities. It is now proposed to briefly review these problems then see how they relate to checking Monte Carlo Simulation Models.

Anderson (1974) suggests that model checking involves two distinct activities namely verification and validation.

(a) Verification. Verification consists of testing the inherent accuracy and internal consistencies of the model. At the lowest level this involves debugging the program and testing the stochastic generators.

Conway (1963) identifies two major problems in verifying computer simulation programs. Both are concerned with the efficiency of execution. The first problem involves measuring the performance of the model at equilibrium. Generally simulation models require "warm up" periods before the performance of the model can be assessed. The second problem identified by Conway is the variability in results due to sample size and the estimation of the precision of the results. Naylor (1971) points out that precision increases at a slower rate than increases in sample size, that is stochastic convergence is slow. For example, to reduce the random error in a sample by a factor of 10 the sample size must be increased by a factor of 100. Gordon (1969) suggests that it may require up to 2000 runs to find the real frequency distribution of the response variables, although this is likely to be an extreme view. Most simulations are carried out with 200 to 500 runs.

(b) <u>Validation</u>. The second activity in checking the model is validation. Here the aim is to test whether the model is a realistic proxy for the real world.

Naylor and Finger (1967) approach the problem of validating computer simulation models in three stages.

(i) The first stage is to look at the rationale or *a priori* assumptions underlying the model.

(ii) Secondly, attempts should be made to verify, empirically, one or more of the assumptions underlying the model, if data are available. (Under Anderson's classification these two stages would come under verification).

(iii) The third stage consists of testing the model against actual or historical values and also its performance in forecasting or predicting the future. Naylor and Finger stress this last stage as being the most important, but consider the three stage approach to be most useful under the following conditions: Firstly, where it is extremely costly or impossible to observe real world processes and secondly where the system is so complex that analytical techniques cannot provide solutions.

(c) <u>Checking Monte Carlo Models</u>. Efforts to validate capital projects under stages two and three of the Naylor Finger procedure are very difficult. Firstly, testing the simulated endogenous variables against historical data is inappropriate. It should be clear that in ex ante evaluations each new project is unique and cannot be compared with the performance of previous projects. Secondly, it is impossible to compare the results of the computer model with that of the actual situation as capital projects generally take a number of years to reach equilibrium. This negates efforts to validate the model through forecasting.

From a practical point of view agencies rarely have the resources available to carry out ex post studies. Also the lack of sophistication in the original ex ante studies make valid comparisons difficult. The data requirements, techniques and documentation of earlier reports is generally not as rigorous as is accepted as normal today.

In Monte Carlo Methods the basis of verification lies in checking the logical relationships that underly the model. Validation becomes unnecessary except where endogenous stochastic input variables are generated. If these dynamic aspects are eliminated then validation by empirical means is no longer necessary.

Perhaps the only useful test appropriate is to compare the output of the simulation model with that of an analytical model. In this way the simulation can be tested for internal consistencies at least.

The above test combined with stage one of the Naylor Finger approach will form the basis of the validation to be carried out on the model in this thesis.

4.6 SUMMARY

Monte Carlo Simulation offers two major advantages over analytical methods. These are firstly, all the characteristics of the probability distribution of the variables can be simulated and secondly the dynamic aspects of project development can be incorporated into the analysis.

While being a theoretically better method of calculating risk and uncertainty there are certain drawbacks to implementation. The major disadvantage is the time required to build a simulation model. Assuming a general program package is not available then a new program must be written for each new project. The task of developing a general package was found to be beyond the scope of this thesis. Further, only a very simple and basic attempt at Monte Carlo Simulation of a particular project was possible within the time available. The main problem encountered was in defining and incorporating the dependency relationships between variables and over time.

This now completes the review of simulation. In the next chapter a particular project is analysed to compare in practice the mathematical programming technique of chapters three and four with the traditional methods of chapter two.

CHAPTER V

COMPARISON OF THE MODELS

5.1 INTRODUCTION

This chapter outlines and presents results for five basic models of risk analysis in project evaluation. They are as follows: 1. Multiple Cases. 2. The Analytical Technique incorporating Taylor's Approximation. 3. Simulation by Monte Carlo Methods using the Multivariate Normal Distribution, and 4. Simulation by Monte Carlo Methods using the Hertz Type Model.

In the initial comparison only the product prices are assumed to be stochastic, all other variables are assumed to be known with certainty. This is followed up by a more rigorous analysis using the Analytical Technique to explore the effect on the variance of introducing uncertainty into per animal performance and total stock increases.

A simple project of a stock water supply scheme is used as an example. The original economic report (Bell, 1971) is presented in Appendix (I). Briefly, the scheme involves a proposal to reticulate for stock water approximately 32,800 hectares (82,000 acres) of rolling to broken farmland in South Otago. Ninety-six properties are involved in the scheme. The major benefits are assessed to be the extra revenue due to extra carrying capacity because of the better water supply.

The costs are due to the installation of the system and the capital and associated costs of the increased stock numbers. Details of the costs and benefits are given in the report cited above.

Although a weighting for overseas funds was introduced in the original analysis this will be ignored in the risk analysis.

5.2 THE MODELS

5.2.1 Multiple Cases Analysis

This model evaluates the project at three levels of prices: optimistic, most likely and pessimistic. These prices represent subjective estimates of "medium term" price ranges at about the 5 and 95 percentile limits. They are generally skewed to the right (i.e. the mean is to the right of the mode). The results are presented using the following three criteria: Present Value, Benefit Cost Ratio, and Internal Rate of Return. A sensitivity analysis of the discount rate is included in the analysis.

Multiple Case Analysis is the approach used in the original report and the detailed results may be found in Appendix I . The results of the project at the ten percent discount rate are shown below in Table 3.

Table (3) Results of Multiple Case Analysis at 10% Discount Rate

	Low	Modal	High
Present Value	$304,253	$1,042,774	$2,006,807
Benefit Cost Ratio	1.09	1.24	1.38
Internal Rate of Return	14.11%	14.11%	26.89%

5.2.2 PERT - Beta Analysis

This is the method suggested by Shepherd (1970) and outlined in section 2.4.5. The results are presented as the expected present value and standard deviation. These are derived directly from the results of the multiple case analysis using equations 1 and 2 .

$$E\ (PV) = \frac{304253 + 4(1042774) + 2006807}{6}$$

$$= \underline{\$1,080,360}$$

$$SD\ (PV) = \frac{2,006,807 - 304,253}{6}$$

$$= \$283,759$$

5.2.3 The Analytical Technique Incorporating Taylor's Approximation. (ANWA).

Being a more rigorous analysis than previous methods the analytical technique requires additional data and assumptions to be made. The following information is required.

(a) The Number of Stochastic Variables to be Included. A decision must first be made on which variables in the analysis are to be treated stochastically. Initially only product prices will be dealt with so that a comparison may be made with other techniques. Later this will be progressively expanded to include per animal performance and total stock increases. As a general rule the variables chosen for stochastic analysis would be determined by a previous sensitivity analysis. Those having a minor effect on present value being ignored.

(b) Probability Distributions. Each uncertain variable is described by a subjective probability distribution. All prices are assumed to have triangular distributions. As indicated in section 3.2.3 the triangular distribution has desirable characteristics, in that it is uniquely defined by its range and mode, can be skewed, and is mathematically tractable. The prices used in this analysis are subjectively derived taking into account such information as historical trends, present conditions, and medium term projections. They are detailed in Appendix II .

(c) Dependencies. The relationships between the stochastic variables within and between periods are very difficult to detect and place estimates on. In this thesis the correlation coefficients are derived by time series analysis.

The auto-correlation coefficients are derived using the relationship

$$\rho_{tt'} = \phi_1^{|t - t'|} \qquad \ldots\ldots\ldots (64)$$

and the first difference equation (45). Cross correlation coefficients are derived from equation (55) which gives the Pearsonian simple correlation coefficients.

The data used to derive the coefficients comes from National Statistics over the years 1956/57 to 1972/73. A computer program was written to derive the correlation coefficients. This program along with the input data and the output coefficients may be found in Appendix III.

(d) The Partial Derivatives. When the cash flows are made up of complex combinations of random variables (or the variables are weighted by deterministic variables) then the partial derivative associated with each stochastic variable must be determined. The method used to derive partial derivatives has been outlined in section 3.1.2 (d). Appendix IV shows in detail how the first derivatives are derived for the example project.

(e) The Mean Cash Flows. Having obtained the means of the stochastic variables these must be combined with the deterministic variables to derive the mean cash flows. The aggregate mean cash flows of benefits, associated costs and capital costs can then be discounted to find the mean present value and internal rate of return. These aggregate flows are also used to find the benefit cost ratios of total benefits to total costs and benefits net of associated costs to capital costs.

(f) Deriving the Overall Probability Distribution of the Present Value and IRR. Having obtained the expected net cash flow, the standard deviations and first derivatives of the stochastic cash flows, and the correlation coefficients (within periods and between periods) the variance of the expected present value can be calculated. This is done using equation (30). The probability distribution of the IRR is calculated using the technique outlined in section 3.1.1 (c).
The layout of the computer cards for input into the program is given in Appendix VI.

(g) Results. The ANWA computer program along with the input data may be found in Appendix V. The results are summarised in Table (4).

The program also calculates and graphs the cumulative probability distribution of the IRR. This graphical presentation can be used to highlight different assumptions as to the interrelationships between the stochastic variables.
Figure (6) illustrates the effect of assuming full dependence

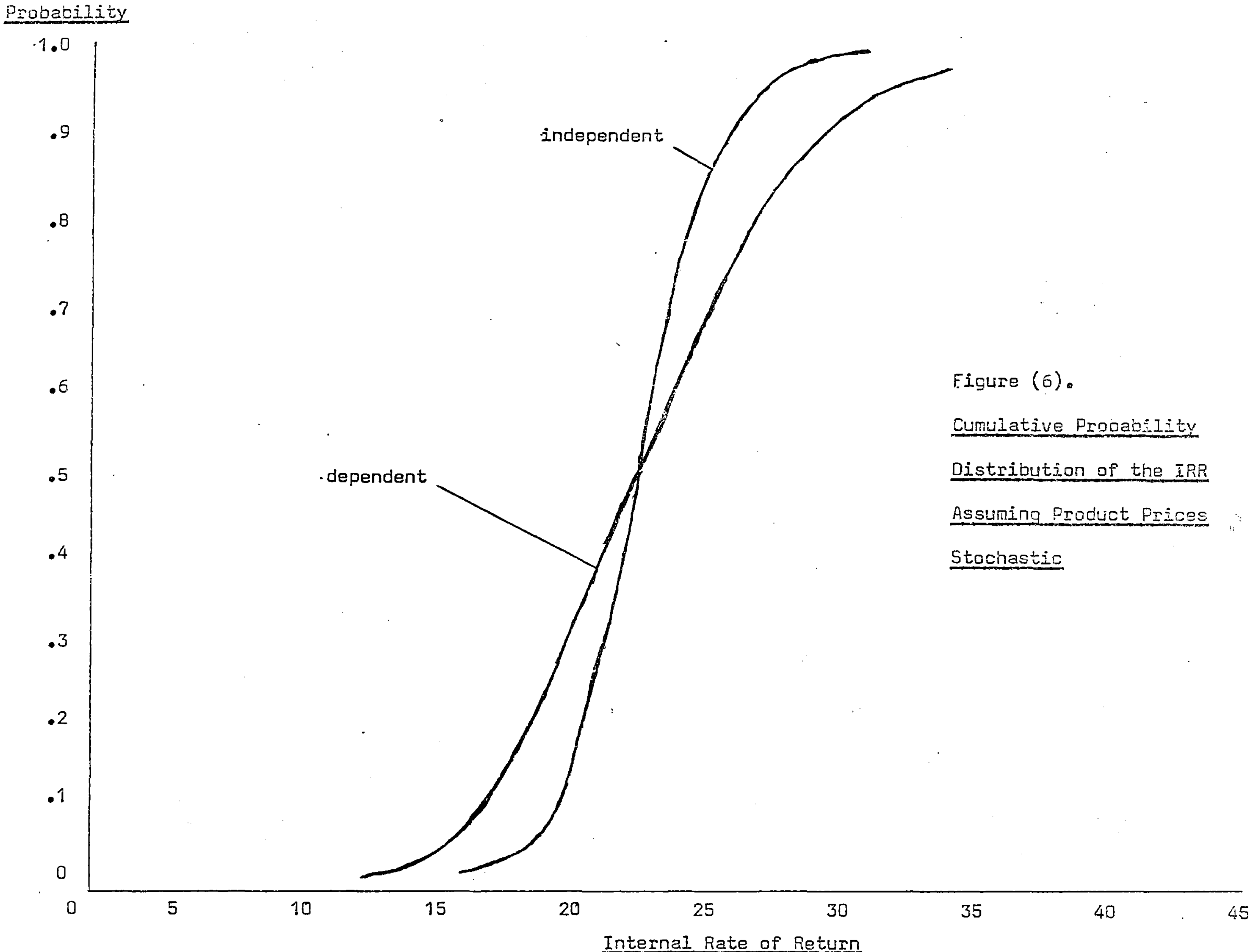


Figure (6).

Cumulative Probability Distribution of the IRR Assuming Product Prices Stochastic and independence on the cumulative probability distribution of the IRR. The standard deviation of the present value is \$139,639 if all the stochastic variables are assumed to be independent, however, if all the variables are assumed to be fully dependent then the standard deviation is \$325,650. This highlights the inadequacies of techniques which ignore or assume away dependency relationships.

Table (4) Results of a Risk Analysis Using ANWA and Assuming Product Prices Stochastic

i	Discount Rate	10%
ii	Expected Present Value	\$1,250,753
iii	Internal Rate of Return	22.5%
iv	Benefit cost ratios	
	- Benefits to total costs	1.28
	- Benefits net of associated costs to total costs.	1.46
v	Standard Deviation of P.V.	\$138,238
vi	Percentiles -5%	\$979,786
	-95%	\$1,521,678
vii	Probability of P.V. .GT . Zero	100%

5.2.4 Monte Carlo Simulation Using the Multivariate Normal Distribution (MVSM)

The method used to simulate samples from the multivariate normal distribution has been outlined in section 4.5.3 and the flow chart which outlines the logic of the computer program is presented in figure (5). The program, input data and output of the model may be found in Appendix VII. The results of the analysis are presented in Table (5).

Table (5) Results of a Risk Analysis Using Simulated Samples from the Multivariate Normal Distribution Assuming Product Prices Stochastic

i	Discount Rate	10%
ii	Number of Simulations	100
iii	Expected Present Value	1,248,740
iv	Standard Deviation	171,849
v	I.R.R.	22.3%

Although it is possible to incorporate more complex combinations of stochastic variables into this type of analysis the program must be changed in order to do so. Apart from the extra computing time required for simulation models this is the major drawback to their implementation on a general basis. This is not so with the analytical techniques.

5.2.5 Monte Carlo Simulation (after Hertz) (TUAP)

Unfortunately the time limitation has not allowed a full exploration of the advantages offered by the Monte Carlo approach. However, this initial attempt has at least highlighted the problems associated with it. The main features of the model are as follows.

(a) Probability Distributions. Triangular density functions are used to represent the distributions of the product prices.

(b) Dependency Relationships. Where the variables are assumed to move together within a period then the dependency or correlation is handled by manipulating the seeds to the stochastic generators. For example, in the model it is assumed that all the cattle prices within a particular period move together. To achieve this in the program, the seeds to the stochastic generators for cattle prices are the same within a year. This means that the generated prices are drawn from the same position in each distribution. The effect is that if steer prices are low then weaner prices will be low along with all other cattle prices within the period. This procedure is similar to the concept of keying as outlined by Kryzanowski et al., op. cit.

All variables are considered to be independent except for the following cases.

(i) The capital cost of sheep is assumed to be dependent on the value of wool and lambs in that period.

(ii) The sheep meat prices of lamb and mutton are assumed to move together.

(iii) All cattle prices are assumed to move together.

This is a very simple and basic model. The literature on Monte Carlo Simulation tends to under-estimate the difficulties of incorporating complex interrelationships into the models. For example, it is difficult to incorporate dynamic time dependent relationships into a general model. For instance, it could be hypothesised that the development rate is a function of revenue:- if prices in year t are high then more revenue is available for investment in year t+1, which will be rewarded by increased benefits in year t +2 etc. Complex procedures are necessary to incorporate these types of relationships when there are a number of products being produced in the scheme.

Also the experience here has shown that a general Monte Carlo Risk package such as has been developed using analytical techniques would be very large and expensive. This is caused by the nature of the method where the functional relationships between variables must be inserted directly into the model. As each new project generally has different functional relationships between variables this means that the basic program must be rewritten for each new project. A general package would have to be sophisticated enough to incorporate all the possible combinations of functional relationships between variables. This was found to be beyond the scope of this thesis.

(c) Results. The program along with input data and an example output run of 500 simulations may be found in Appendix VIII. A summary of the results for 500 simulation runs is given in Table (6).

Table (6) Results of a Monte Carlo Risk Analysis

(i)	Discount Rate	10%
(ii)	Number of Simulations	500
(iii)	Expected Present Value	$1,262,435
(iv)	Standard Deviation	$ 67,328
(v)	I.R.R.	22.72

5.3 THE MODELS COMPARED

5.3.1 Results

A summary of the results is presented in Table (7) below.

Table (7) Summary of Results

	PERT	ANWA	MVSM	TUAP
Present Value	1,080,360	1,250,732	1,251,765	1,262,435
Standard Deviation	283,759	138,238	171,649	76,328

From a theoretical point of view the MVSM model is the most sound, being based on samples from normal distributions. We would therefore place the most confidence on the results produced by this model. When compared with the results of the other models there are large differences among the standard deviations. ANWA returns a standard deviation which is 19 percent less than that of MVSM while TUAP is 56 percent less and the PERT standard deviation is 66 percent higher.

The difference between the results of MVSM and ANWA is due to the fact that ANWA uses an approximation to derive the standard deviation. It was pointed out in section 3.1.2. (c) that the variance of a multiple of random variables is an approximation where the relative error depends on the coefficient of variation of the variables. For the product prices these coefficients range from 0.10 to 0.22. Thus the error in individual products will be around two percent. However, with a number of products over many years the errors have tended to accumulate rather than cancel each other out thus producing a lower result.

As expected the standard deviation returned by the Hertz model is very low. This is explained by the difficulties of incorporating realistic dependency relationships.

The results of the multiple cases analysis and the PERT - Beta are not strictly comparable with those of ANWA and MVSM. This is because of the skew introduced into the distribution of total stock increases for the analytical and simulation studies (see Appendix II (c)). However, this has the effect of reducing the difference between the standard deviations of the two types of models. Assuming the skew had not been introduced into the distributions of total stock increases, then the standard deviations would be further apart still. From the theoretical background of section 3.2.3 we would have expected a very high standard deviation in the PERT model. This has been borne out by the results and provides analytical evidence for rejecting the PERT approach to risk analysis.

5.3.2 The Relative Difficulties Associated with Implementing the Models

(a) Data Requirements. As the models become more sophisticated the time and cost of preparing data increases. However, there are certain data requirements common to all models. These are the values for all the deterministic variables over all years and the estimates of the parameters which define the probability distributions of the stochastic variables. The Multiple Case and PERT - Beta require no additional information.

Both ANWA and MVSM require in addition to the above, estimates of the correlation coefficients for dependencies between and within periods. At the most sophisticated levels Hertz type models require the analyst to be explicit about dynamic aspects of the project.

(b) Data Processing. The time required to prepare and code the input for the computer is roughly equivalent for all the models except for the Hertz model. The Multiple case and PERT-Beta Analysis require the deterministic cash flows, and the uncertain cash flows to be calculated at three levels - high, modal and low. On the other hand ANWA only requires the cash flows to be estimated at their expected levels, but it also requires the flows of standard deviations and the correlation coefficients. MVSM requires the means and standard deviations of the uncertain variables rather than the actual cash flows. However, when the random variables form products with deterministic variables these must be coded as flows.

After an initial period of development and testing most of the above information could be standardised and used universally in all projects. The type of information would include the parameters of the probability distributions of the prices and the correlation coefficients.

The Hertz model presents more of a problem. The model is built up of components which interact by the functional relationships including dynamic aspects. The advantage of simulation is that these components etc are often unique to a particular project thus highlighting its particular characteristics. However, this means that the computer program must be rewritten for each project - a costly and time consuming business. From the experience built up in this study it is my conviction that the extra effort is not justified by the results.

(c) <u>Computer Time</u>. Table (8) below sets out the relative times required by each model in the Central Processing Unit (CPU) of the B6700 computer. These figures highlight a major disadvantage of the simulation techniques - the cost of computation.

Table (8) <u>Computer Time Required by each Model</u>

<u>B6700 Computer</u>

		Seconds
i	Multiple Case	4
ii	ANWA - without graph of IRR	6
	- with graph of IRR	372
iii	MVSM	688
iv	TUAP (Hertz)	360

The above results are taken using 100 simulation runs for the Monte Carlo Models - a barely adequate number of runs.

5.4 CONCLUSIONS

Although the Analytical Technique incorporating Taylor's Theorem gives an approximation of the standard deviation the advantages of the technique outweigh this deficiency. Also, when compared with the other models the analytical method does not have the practical problems of implementation.

Monte Carlo simulation models suffer generally because of the considerable amount of computer time required to obtain results. Peculiar to the multivariate normal simulation technique is the problem of size of matrix. It is theoretically possible to incorporate both cross correlation and auto-correlation into the generating procedure. In practice the size of the matrix becomes too large. For example, the price analysis carried out here has 10 stochastic variables with a project life of 36 years. This would require a matrix of 360 by 360. If all 25 variables are included, the matrix size is 900 by 900. In terms of computer storage this occupies 1.62 million words of space. Another problem with this method is that reprogramming is necessary when different types of variables (products versus sums) are included in the analysis.

The major problem with the Hertz type Monte Carlo Model is incorporating the dynamic dependency relationships into a program which is general enough to handle the normal variations between different projects.

A major problem of all probabilistic models is obtaining the parameters of means, standard deviations and correlation coefficients. However, once derived they can and should be used on a national basis for consistency of analysis.

The advantages of using probabilistic models has already been outlined (see section 3.4). The specific advantages of ANWA over the other models are as follows:

(a) The program is general and can handle any complex combinations of variables with up to 30 stochastic flows of up to 50 years without changing the program.

(b) Once the basic parameters have been decided on the time required to set up the input is about the same as the Multiple Case Analysis.

(c) The program is relatively cheap on computer time, requiring only several seconds if a graph of the cumulative probability distribution of the IRR is not required. Where a graph is required CPU time ranges from 375 to 700 seconds where the number of stochastic variables range from 10 to 25.

The fact that ANWA may under or over estimate the standard deviation is not critical as long as all the projects are analysed under the same set of assumptions and techniques. Risk analysis promotes consistency of analysis and it is this point which is most critical to making good decisions on the allocation of scarce resources.

5.5 IN DEPTH ANALYSIS USING ANWA

Apart from product prices there are two other major sources of uncertainty in agricultural type projects. These are the productivity levels of the introduced activities and also the absolute levels reached by the activities. ANWA is able to incorporate both these aspects to gauge the effect on the variance of the present value. The results of the analysis are presented in Table (9) and the details of the probability distributions of the productivity levels and total stock increases are presented in Appendix II. A comparison is also made between the assumptions of all variables being independent and all variables dependent.

Table (9) The Effect on the Standard Deviation of Increasing the Number of Variables Within the Risk Analysis

Dependency Relationship	Independent	Calculated level of Dependency	Fully Dependent
(a) Product Prices Stochastic	$139,639	$138,238	$325,650
(b) Prices and Quantities Stochastic	164,365	140,359	528,068
(c) Prices, Quantities and Stock Increases Stochastic	185,590	208,310	597,090

In the initial evaluation of product prices there are 10 stochastic variables, with productivity levels stochastic there are 19 variables and with total stock increases stochastic as well there are 25 variables.

From Table (9) it can be seen that as the number of variables in the analysis increases the standard deviation also increases. This is as would be expected. Unexpectedly, perhaps, the effect of altering the dependency relationships within the analysis is not quite as straight forward. In both the price analysis and the price quantity analysis, moving from the situation of full independence to the calculated level of dependency actually causes the standard deviation to fall. This is generally quite unusual. One possible explanation is the fact that a number of the calculated correlation coefficients are negative (see Appendix III). Where this occurs the effect is to reduce the standard deviation.

An obvious conclusion from these results is the overriding influence of the level of assumed or calculated correlation between variables on the standard deviation. For this reason the analysis of dependency between variables should receive a very high priority if the technique is to be of any value.

5.6 POSSIBLE IMPROVEMENTS TO THE ANWA PROGRAM

Several improvements to the present ANWA model are possible to reduce the number of calculations required to obtain the data for input.

(i) At present the model uses as input the net flows of expected benefits, associated costs, and capital costs. This requires the analyst to aggregate the various cash flows by hand. To overcome this it would be worthwhile binding ANWA to the present discounting routine used by the Division which has the facility of feeding in the disaggregated flows.

(ii) The weights or first derivatives required to calculate the standard deviation must also be calculated by hand. This is one area where extra computation is necessary and is therefore an area where human errors could occur. To overcome this problem the different weights could be derived within the program using a series of transformations. Transformation procedures are a feature of most multiple regression packages, see for example AJOLS (Rodgers, 1973) or AWERI and AWER2 (Woods, 1973).

(iii) As the basic data requirements are the same for deriving both the expected net cash flows and the weights points (i) and (ii) above could be incorporated into the program together much reducing the initial computational burden to obtain input data to the program.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 THE PROBLEMS OF IMPLEMENTING RISK ANALYSIS

Just as discounting gained acceptance as a technique in the mid 1960's risk analysis is now gaining acceptance in the mid 1970's. Although operations researchers tend to simplify the implementation of the technique several writers have pointed out that it may not be so straightforward.

Bourke (1969) points out that the major difficulties in implementing risk analysis are psychological rather than mathematical. It is in the data evaluation stages that most of the difficulties occur. These are in delineating the alternative possibilities and then estimating the probability of their occurrence. Brown (1970) surveyed 20 companies in the United States to assess their experiences as practitioners of risk analysis. He found that the technical problems of implementation are not the most serious or the limiting factors. It is the promotion of the concepts, the way the techniques are presented and the role or importance of risk analysis in the overall decision making area which requires the most emphasis for the maximum advantages to be gained. Carter (1972) looked at the conditions which helped or hindered implementation of risk analysis in four major oil companies. He found several inherent problems; the most important are as follows:

The relevance of risk analysis must be made clear to those involved at all stages. There are problems of obtaining data assessments. People must be educated to think in terms of probabilities. Decision makers had problems in trading off the risk and return. Managers wanted the analysts to give a lead on defining utility functions instead of making their own tradeoffs. Other problems identified by Carter include the evaluation of ex post studies, aligning problems with models and the human and organisational problems of introducing a new technique. Pouliquen (1970) concludes that risk analysis should only be undertaken with the greatest caution. His major concern is the completely misleading results that can occur if correlations between variables are not handled correctly.

6.2 THE ADVANTAGES OF RISK ANALYSIS

Notwithstanding the problems above, risk analysis using the analytical technique incorporating Taylor's approximation is a feasible and desirable alternative to traditional methods. The advantages may be summarised as follows:

(i) The Technique utilizes a great deal of information which would otherwise be lost.

(ii) The framework of analysis offers a medium of communication as a focus for discussion among involved parties.

(iii) Attention is focused on the expected present value rather than modal estimates.

(iv) The technique provides an explicit statement of the uncertainty rather than an implicit one.

(v) The variability of the project is measured by a simple criterion, the standard deviation rather than a number of different criteria.

(vi) Judgment is applied to the underlying assumptions rather than to the results.

(vii) Allowance is made for interaction between the variables in the project thus promoting a more rigorous analysis.

(viii)Most importantly the method provides a framework for analysing projects in a consistent way, analyst to analyst and project to project thus allowing valid comparisons between projects for decisions on the allocation of scarce capital resources.

6.3 IMPLEMENTATION

Before implementing the suggested risk analysis technique several considerations should be taken into account. The first and major task which should be undertaken is to look more closely at the parameters which form the basis of the analysis. These are the estimates of the means, standard deviations and correlation coefficients. In the time available it has been only possible to have a perfunctory look into these factors yet the whole credibility of the analysis rests on them. The most critical are the correlation coefficients. As was shown in section 5.5 they exhibit an over-riding effect on the standard deviation. In section 3.3 several techniques are outlined which could be further developed. Also the data used in the correlation analysis is of a very gross form. Further research would be desirable into such avenues as regional differences.

Obviously, the technique requires testing on a series of typical projects to iron out any difficult areas. Before this could be done it would be essential to gather together the technical information related to implementing the analysis into a user manual.

Another important aspect of implementation is the training of analysts (and those they consult for information) in the concepts of subjective probabilities. This is an area which is still in flux (see section 3.2). Only experience will sort out a consistent workable method.

Lastly, it would be desirable for other sectors of the economy, such as Forestry and Works, which compete for the same resources to co-operate in evaluating joint proposals. The risk analysis technique provides a forum for such discussion.

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No responsibility for any inconsistencies in this thesis attaches to those mentioned above.

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APPENDIX I

ECONOMIC REPORT
ON THE
TUAPEKA RURAL WATER SCHEME
NOVEMBER 1971

by B.A. Bell,
Agricultural Economist,
Department of Agriculture,
PALMERSTON NORTH.

CONTENTS

INTRODUCTION

SECTION I

(1) Description of the Proposed Scheme

The proposed Tuapeka Rural Water Supply Scheme has been designed to reticulate an area of approximately 82,000 acres of farmland in South Otago. The area is roughly triangular and is bounded to the north by Beaumont to the south by Tuapeka Mouth and to the east by Waituhuna.

The sources of water are the Tuapeka and Waitahuna rivers.

Water will be delivered to each consumer's tank or tanks on his property at a constant flow rate over 24 hours each day, so that each consumer receives a predetermined number of units of water, each unit being approximately 400 gallons per day. It will be the farmer's responsibility to reticulate his property from the tank position, and to provide the necessary troughs and fittings.

Details concerning the engineering aspects of the scheme may be found in a report prepared by G.T. Gillies Limited, Electrical, Hydraulic and Mechanical Engineers.

A list of farmers subscribing to the scheme was provided by the Tuapeka County Council, 96 properties are involved. Physical data was obtained by an interview survey of a random sample of these properties. The sample contained 40 properties.

Those farmers who did not apply for water are not considered in this analysis.

(2) Summary of Results

At the medium or expected level of costs and prices the results of the Tuapeka Rural Water Scheme are as follows:-

TABLE 1 : SUMMARY OF RESULTS

A. Unweighted

Interest Rate	Present Worth	Benefit/Cost Ratio
9%	$1,256,247	1.27
10%	$1,042,774	1.24
11%	$863,142	1.22

Internal Rate of Return 20.92

B. Weighted for Overseas Funds Content

Interest Rate	Present Worth	Benefit/Cost Ratio
9%	$1,568,952	1.44
10%	$1,317,861	1.42
11%	$1,106,210	1.31

Internal Rate of Return 23.10

(3) Prices Used : On-Farm

Pipe ½"	alkathene	2.15 per chain
¾"	"	4.30 " "
1"	"	5.19 " "

Troughs 85 gal. $15.40 + $4.50 for fittings

200 gal. $23.00 + $4.50 for fittings

Tanks 600 gal. $77.00

3,000 gal. $229.00 (includes 40 miles travel at 50c)

5,000 gal. $358.00 (includes 40 miles travel at 75c)

Fencing - Cyclone boundary netting with 1 barb $12.50 chain

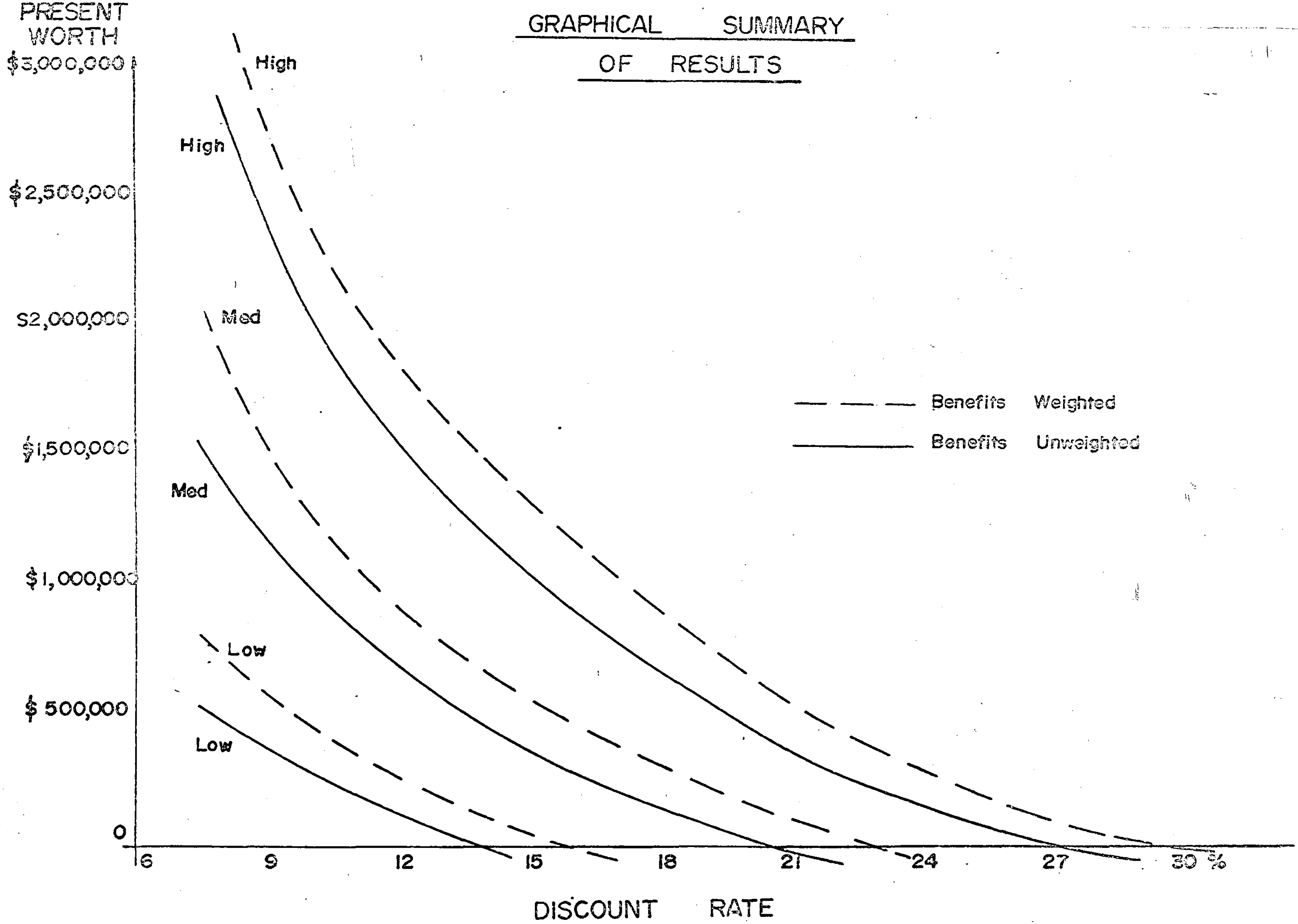
Seven wire $12\frac{1}{2}$ gauge with 1 barb $12.00

Fertiliser - Superphosphate $30 per ton -spread.

Capital Stock

		Per Head
Sheep	Lowest	$ 4
	expected	$ 6
	highest	$ 10
Beef Breeding	L	$ 60
	E	$100
	H	$140
Beef Fattening	L	$ 35
	E	$ 70
	H	$110

GRAPHICAL SUMMARY OF RESULTS



SECTION II

ECONOMIC ANALYSIS

(1) Method

The method used in this analysis has been to compare the agricultural potential of the area with and without the scheme. Any differences which occur in the value of production or in on-farm physical inputs are translated into monetary terms, for the appropriate years, and then the cash flows are derived for the benefits and costs. Off-farm capital and running costs are treated in a similar way. Finally, all cost and benefit streams are discounted. The cost and benefit flows can be found on pages 17 to 20.

The results are presented using three criteria:

Present Worth.

Benefit Cost Ratio.

Internal Rate of Return.

Separate results are presented showing the effect of a 10% premium on the overseas funds content of costs and benefits on the above criteria.*

Finally, three levels of prices and costs were used to show how sensitive the results are for prices and costs: A medium or expected level, which is the best estimate of the forseeable future and a high and low estimate of what could possibly happen.

The scheme life has been estimated to be 36 years.

* Cost Benefit Analysis and Overseas Earnings by the Resource Economics Section, Economics Division, Department of Agriculture, Palmerston North.

(2) Costs

(a) Capital Costs - off-farm and on-farm costs incurred in capital construction of the scheme.

(b) Associated Costs - off-farm and on-farm costs incurred in operating and maintaining the scheme.

(3) Benefits

(a) Tangible Benefits

Those benefits to the scheme that can be measured in monetary terms.

(i) Production Benefits -

The extra monetary returns from the expected extra stock carried because of the scheme. It is expected that, with the scheme, stock numbers will increase from 3.9 to 4.7 S.U. per acre. Without the scheme the expected increase is to 4.2 S.U. Thus there is an expected increase of 0.5 S.U. per acre due to the scheme.

TABLE II

EXTRA STOCK CARRIED DUE TO THE SCHEME (S.U.)

(Sampled Farms)

Year	Sheep	Beef Breeding	Beef Fattening	Cumulative Increase
2	3525	1580	1334	6439
3	2820	1053	890	11202
4	705	1053	890	13850
5		1053	890	15793
6		528	444	16765

(ii) Saved On-farm Costs -

These are the savings in costs that would have to be incurred to provide a water supply if the proposed scheme is not implemented. See Appendix 3 for details.

(b) Intangible Benefits

Those benefits which are indeterminate or too difficult to measure in economic terms.

The main benefits will be improved stock and grazing management through the summer period and the ability to water cattle with safety.

4. Results and Conclusions

The results of the scheme are detailed in Table III.

It can be seen that under all combinations of price levels and discount rates the project is profitable to the nation when measured by any of the three criteria used in analysis, i.e., Present Worth, Benefit/Cost Ratio on Internal Rate of Return.

TABLE III

RESULTS

(A) Unweighted	Interest Rate 9%	10%	11%
Low Level			
Present Worth (P.W.)	$ 415,863	$ 304,253	$ 210,613
Benefit Cost Ratio (B/C)	1.11	1.09	1.06
Internal Rate of Return (I.R.R.)	14.11		
Medium Level			
P.W.	$1,256,247	$1,042,774	$ 863,142
B/C	1.27	1.24	1.22
I.R.R.	20.92		
High Level			
P.W.	$2,350,865	$2,006,807	$1,716,876
B/C	1.41	1.38	1.35
I.R.R.	26.89		

(B) Weighted for Overseas Funds Content

	9%	10%	11%
Low Level			
P.W.	$ 592,807	$ 454,363	$ 335,599
B/C	1.17	1.16	1.12
I.R.R.	16.12		
Medium Level			
P.W.	$1,568,952	$1,317,861	$1,106,210
B/C	1.44	1.42	1.31
I.R.R.	23.10		
High Level			
P.W.	$2,760,107	$2,366,396	$2,034,419
B/C	1.50	1.47	1.44
I.R.R.	28.75		

SECTION III

AGRICULTURE IN THE AREA

(i) General

The present pattern of farming in the area is mainly sheep with supporting beef cattle and a very limited amount of cash cropping.

The country varies in contour from easy rolling to steep. It is often broken by deep gullies. The rainfall is from 25-40" and is fairly well distributed although a summer dry period does occur. However, the natural water supply is poor with few permanent streams.

(ii) Soils

Soils vary from yellow-grey earths through to lowland yellow-brown earths. The main soil types are: Kononi soil (silt loam), Waitahuna soils (silt loams), Waitahuna Hill Soils (silt loams and stony silt loams), Tuapeka Steepland Soils (silt loams and stony loams), and Tuapeka Hill Soils (silt loams to stony loams).

(iii) Water Supply

(a) Farm

At the moment water supply is probably the limiting factor to increased production. Water shortage in the summer causes management problems. Gates must be left open to give stock access to water, and existing water becomes stagnant or dirty as dams and streams dry up. Approximately half the farmers thought that water quality was poor for stock and on average farmers were short of stock water for ten weeks during the summer. In a normal season little stock water is purchased.

TABLE IV

WATER SUPPLY (Sampled Farms)

Dams	33
Streams	29
Springs	25
Bores	2
Wells	1
Community Scheme	1

N.B. Some farms have more than one of the above sources.

(b) Domestic

The majority of domestic water supply comes from stored rain water. Approximately 50% those questioned thought that the supply was inadequate but only 18% purchased water in a normal year.

(iv) Production Data

TABLE V

LIVESTOCK WINTERED 1971

(Sampled Farms)

Sheep	No.	S.U.	TOTAL S.U.
Ewes	89645	1.0	89645
Ewe hoggets	25126	0.6	15076
Rams	2433	0.8	1946
Wether hoggets	2411	0.6	1447
Wethers	2091	0.8	1673
TOTAL	121706		109787
Cattle			
Breeding cows	1424	6.0	8544
Replacement heifers	252	4.0	1008
Rsg 1 yr cattle	869	4.0	3476
Rsg 2 yr cattle	541	4.0	2164
Bulls	56	5.0	280
TOTAL	3142		15472
Total Stock Units	125259		
S.U.S. per Effective Acre:	3.9		
Sheep to Cattle Ratio:	39 to 1		

TABLE VI

LAND UTILISATION JANUARY 1971

(Sampled Farms)

	Area Acres	%
New Grass	1233	2.8
Good Pasture	24378	55.4
Poor Pasture	4744	10.8
Turnips	432	1.0
Swedes	421	1.0
Rape	267	0.6
Chou Moellier	171	0.4
Kale	124	0.3
Wheat	74	0.2
Ryegrass	200	0.5
Unimproved Area	11698	26.6
Waste *	171	0.4
	43913	100.0

* Waste includes yards, plantations, buildings, etc.

Effective acreage: 32,044.

SECTION IV

APPENDICES

(1) Costs Used

(A) Off-Farm Costs

Capital	- Pipelines		$110,678
½ cost of	- Rising Main Waitahuna	$21,540	
	- Waitahuna Pumping Station	$24,000	
	- Treatment Station	$34,700	
		$80,240	$ 40,120
	Pipe Fittings, etc		$ 64,400
	Break Tanks		$ 1,600
			$216,798
	Contingencies 10%		$ 21,679
			$238,477
	Escalation of Costs 5%		$ 11,900
			$250,377
	Engineering Fees 6.3%		$ 15,700
			$266,077
	Voluntary Labour		$ 12,000
	TOTAL CAPITAL COST		$278,077
Associated	Electric Power		$ 3,903
	Chemicals		$ 1,300
	Maintenance		$ 1,500
	Pump and Treatment Plant Replacement		$ 1,200
	TOTAL ASSOCIATED COSTS		$ 7,903

(B) On-Farm Costs

The prices used are shown on page 2.

GROSS REVENUE AND COSTS (TO SCHEME) OF SHEEP PER 1,000 EWES (Showing weightings for Overseas Funds)

Assumptions: 105% lambing ewes tupped to sale age, 3% ewe deaths. 1% ewes culled, 1 ram to 40 ewes

					Overseas Funds Content	Overseas Funds Content Weighted by 10%		
REVENUE TO F.O.B.		low	expected	high		low	expected	high
1. Wool - ewes 970 at 10 lb)	44c/kilo FOB	2,420						
- ewe lambs 219 at 3 lb) 12126 lb at	55c " "		3,025					
- hoggets 217 at 7 lb) (5500 kilo)	89c " "			4,895				
- rams 25 at 10 lb)								
2. Meat - fat lambs 831 at 30lb carcass wt	19c/lb FOB	4,737						
	25c " "		6,233					
	38c " "			9,473				
- cull for age ewes 187 at 50 lb carcasswt at	10.5" "	982						
	13 " "		1,216					
	18.5" "			1,730				
Total Revenue to F.O.B.		8,139	10,474	16,098	100%	8,953	11,521	17,708
COSTS FARM GATE TO F.O.B.								
1. Wool - processing 5500 kilo at	10.5c/kilo	578						
	11 "		605					
	12 "			660				
2. Meat - lamb processing at	6c/lb	1,496	1,496	1,496				
- cull ewe processing at	5c/lb	468	468	468				
Total Off-Farm Costs		2,542	2,569	2,624	9.1%	2,565	2,592	2,648
Income to Farm Gate		5,615	7,905	13,474		6,388	8,929	15,060
ON-FARM VARIABLE COSTS								
Shearing and Crutching at 60c per ewe			600		9.2%		606	
Shed expenses cartage			152		9.2%		153	
Animal Health at 48 cents per ewe			480		9.2%		484	
Rams			300		100%		303	
TOTAL ON-FARM VARIABLE COSTS		1,532	1,532	1,532		1,546	1,546	1,546
GROSS MARGIN per 1000 ewes (1150 ee)		$4,065	$6,373	$11,942		4,842	7,383	13,514
per 1000 ewe equivalents		$3,535	$5,542	$10,384		4,210	6,420	11,751

GROSS REVENUE AND COSTS (TO SCHEME) OF BEEF BREEDING per 100 cows (showing weighting for Overseas Funds)

Assumptions: 3% cow deaths, 3% heifer deaths, 90% calves to weaning.

		low	expected	high	Overseas Fund Content	Overseas Funds Content Weighted by 10%: low	expected	high
REVENUE TO F.C.B.								
45 steer weaners at	$47 FOB value	2,115						
	82 " "		3,690					
	122 " "			5,490				
25 heifer weaners at	32 " "	800						
	67 " "		1,675					
	107 " "			2,675				
15 cull cows 400 lb carcass wt at	19.50 per 100 lb FOB value	1,170						
	28.50 " " "		1,710					
	37.50 " " "			2,250				
0.6 bulls 750 lb carcass wt at	22.50 " " "	101						
	33.50 " " "		151					
	44.50 " " "			200				
TOTAL REVENUE TO F.O.B.		4,186	7,226	10,615	100%	4,605	7,949	11,677
COSTS FARM GATE TO F.O.B.								
Added value of weaners at $12 per weaner			840					
Added value of processing cull cows at $6 per 100 lb carcass wt			360					
Added value of processing bulls at $6 per 100 lb carcass wt			27					
TOTAL OFF-FARM COSTS		1,227	1,227	1,227	9.1%	1,238	1,238	1,238
Income to Farm Gate		2,959	5,999	9,388		3,367	6,711	10,439
ON FARM VARIABLE COSTS								
0.6 bulls at	$200	120			100%	132		
	300		180		100%		198	
	400			240	100%			264
Animal Health at $3 per cow			300		9.2%		303	
Freight on sales			103		9.2%		104	
Winter Feed 29 bales per cow at 30 cents per bale			870		9.2%		878	
Brucellosis			45		9.2%	1,330	45	
TOTAL ON-FARM VARIABLE COSTS		1,438	1,498	1,558		1,462	1,528	1,594
GROSS MARGIN per 100 breeding cows (684.5 ee)		1,521	4,501	7,830		1,934	5,212	8,874
per 1000 ewe equivalents		2,222	6,576	11,439		2,825	7,614	12,964

GROSS REVENUE AND COSTS (TO SCHEME) OF BEEF FATTENING per 100 weaners (showing weighting for Overseas Funds)

Assumptions: 1% deaths, selling 18-20 months

					Overseas Funds Content	Overseas Funds Content Weighted by 10%		
REVENUE TO F.O.B.		low	expected	high		low	expected	high
99 beasts 450 lb carcase at	$21 per 100 lb	9,356						
	$31 " " "		13,811					
	$41 " " "			18,266				
Less F.O.B. value attributable to weaners at $35 + $12	$47 per beast	4,700						
(Price Farm Gate plus processing charge) $70 + $12	$82 " " "		8,200					
$110 + $12	$122			12,200				
Total Revenue to F.O.B.		$4,656	$5,611	$6.066	100%	5,122	6,172	6,673
COSTS FARM GATE TO F.O.B.								
Added value to beef carcase at $6 per 100 lb			2,673					
Less added value attributable to weaner at $12 per weaner			1,200					
TOTAL OFF-FARM COSTS		1,473	1,473	1,473	9.1%	1,486	1,486	1,486
Income to Farm Gate		3,183	4,138	4,593		3,636	4,686	5,187
ON-FARM VARIABLE COSTS								
Animal Health	at $1.50 perhead		150					
Winter Feed	at 12½ bales per head at 30c/bale		375					
Freight on Sales			268					
TOTAL ON-FARM VARIABLE COSTS		793	793	793	9.2%	800	800	800
GROSS MARGIN per 100 weaners (400 ee)		$2,390	$3,345	$3,800		$2,836	$3,886	$4,387
per 1000 ewe equivalents		$5,975	$8,363	$9,500		$7,090	$9.715	$10,968

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CASH FLOW. FARM COSTS ($) UNWEIGHTED (Sampled farms, expected level of costs)

YEAR	0	1	2	3	4	5	6	34	35	36
CAPITAL COSTS										
Pipe 1"	105	26	26	11	11	0				
¾"	12,229	1,746	1,746	873	873	0				
½"	675	99	99	47	47	0				
Troughs 85 gal.	1,831	259	259	139	139	0				
200 gal.	5,115	743	743	358	358	0				
Tanks 400 gal.	77									
3000 gal.	2 748	458	458	229						
5000 gal.	8,950	1,432	1,432	716						
Fences - mesh	3,750	3,750	3,750							
- 12½ gauge	6,048	6,048	6,024							
Stock - sheep		18,400	14,720	3,680						
- beef breeding		23,084	15,384	15,384	15,384	7,714	0			
Total Capital Costs	41,528	56,045	44,641	21,437	16,812	8,000	0			
ASSOCIATED COSTS										
Repairs and maintenance			2,076	2,804	3,531	3,650	3,721	3,721	0	
Fertiliser		7,500						7,500	0	
Sheep			12,566	22,620	25,134				25,134	0
Beef Breeding			6,285	10,474	14,663	18,852	20,952		20,952	0
Beef Fattening			7,558	12,601	17.644	22,687	25,202		25,202	0
Total Associated Costs		7,500	35,985	55,999	68,472	77,823	82,509	82,509	71,288	0

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YEAR	0	1	2	3	4	5	6	34	35	36
Stock Benefits										
Sheep			32,099	57,778	64,197				64,197	0
Beef Breeding			16,669	27,778	38,887	49,996	55,567		55,567	0
Beef Fattening			18,712	31,196	43,680	56,164	62,392		62,392	0
On-Farm Saved Costs										
Capital - dams		5,400	5,400	5,400	0					0
- pumps		1,038	1,038	519	519	519	0			0
Associated - power			435	485	510	535			535	0
- maintenance			180	200	210	220		220	0	0
										0
- dam maintenance			950					950	0	0
- water purchase - stock			200						200	0
- domestic			280						280	0
- labour			2,500						2,500	0
Salvage										
Pumps		390	0							
Sheep									36,801	0
Beef Breeding									76,951	0
Beef Fattening									77,840	0
TOTAL BENEFITS		6,828	78,463	127,286	151,933	175,561	186,841	186,841	377,263	0

YEAR	0	1	2	3	4	5	6	34	35	36
CAPITAL COSTS										
Pipe 1"	106	26	26	11	11	0				
¾"	12,342	1,762	1,762	881	881	0				
½"	681	100	100	47	47	0				
Troughs 85 gal	1,848	261	261	140	140	0				
200 gal	5,162	750	750	361	361					
Tanks 600 gal	78	0								
3000 gal	2,773	462	462	231	0					
5000 gal	9,032	1,445	1,445	723	0					
Fences Mesh	3,785	3,785	3.785	0						
12½ gauge	6,104	6,104	6,104	0						
Stock Sheep		20,240	16,192	4,048	0					
Beef Breeding		25,392	16,922	16,922	16,922	8,485	0			
TOTAL CAPITAL COSTS	41,911	60,327	47,809	23,364	18,362	8,485	0			
ASSOCIATED COSTS										
Repairs and maintenance			2,095	2,830	3,563	3,684	3,755	—3,721	0	
Fertiliser		7,569	—	—	—	—	—	—7,569	0	
Sheep			12,655	22,779	25,310	—	—	—	—25,310	0
Beef Breeding			6,379	10,632	14,884	19,163	21,268	—	—21,268	0
Beef Fattening			7,624	12,710	17,797	22,883	25,421	—	—25,421	0
TOTAL ASSOCIATED COSTS	0	7,569	36,322	56,520	69,123	78,609	83,323	—83,323	71,999	0

YEAR	0	1	2	3	4	5	6	7	[illegible]	[illegible]
Stock Benefits										
Sheep			35,310	63,558	70,620	—	—	—	70,620	0
Beef Breeding			18,337	30,559	42,780	55,001	61,129	—	61,129	0
Beef Fattening			20,583	34,316	48,049	61,781	68,633	—	68,633	0
On-farm Saved Costs										
Capital - dams		5,450	5,450	5,450	0					0
- pumps		1,048	1,048	524	524	524	0			0
Associated - power			439	489	515	540	—	—	540	0
- maintenance			182	202	212	222	—	222	0	0
- dam maintenance			959	—	—	—	—	959	0	0
- water purchase - stock			202	—	—	—	—	—	202	0
- domestic			283	—	—	—	—	—	283	0
- labour			2,523	—	—	—	—	—	2,523	0
Salvage										
Pumps		394	0							
Sheep									40,481	0
Beef Breeding									84,646	0
Beef Fattening									85,624	0
TOTAL BENEFITS	0	6,892	85,316	139,065	166,667	192,655	205,111	205,111	414,681	0

SECTION V

ACKNOWLEDGEMENTS

1. P.W. Vucetich, Farm Advisory Officer, Balclutha for local advice.

2. A.W. Hughes and M.I. Robinson, Agricultural Economists, for aid in survey work.

3. Farmers in the scheme area for co-operation.

4. Typists, Christchurch.

APPENDIX II

PROBABILITY DISTRIBUTIONS OF THE STOCHASTIC VARIABLES

In the analysis three levels of uncertainty are assumed.

(i) All product prices.

(ii) Product prices and productivity levels.

(iii) Product prices, productivity levels and total stock increases.

The probability distributions of the above variables are derived as follows:

(a) Product Prices. All prices are assumed to have triangular distributions where the range is given by the 5 and 95 percentile limits. Table (10) gives the parameters of the distributions (low, modal, and high estimates) together with the code used to identify the variables in the analysis and the derived means (μ) and standard deviations (σ).

(b) Productivity Levels. The productivity variables are assumed to be normally distributed with 5 and 95 percentiles equal to $\pm$ 20 percent from the mean. The variables and distributions are given in Table (11).

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Table (10) Product Price Distributions

	Flow No.	Code	Low	Modal	High	μ	σ
1. Sheep							
capital stock/SU	2	PS	\$4.00	\$6.00	\$10.00	\$6.65	\$1.25
wool/kilo	11	PW	.44	.55	.89	.62	.09
lamb/kilo	13	PL	.42	.56	.84	.60	.08
cull ewe/kilo	15	PM	.23	.29	.41	.30	.03
2. Beef							
Weaner heifer/ kilo	18	PHF	.39	.82	1.31	.83	.18
cull cow/kilo	20	PCL	.43	.63	.83	.62	.08
bull/kilo	22	PBL	.50	.74	.99	.74	.10
fat steer/kilo	24	PST	.47	.69	.91	.68	.09
breeding cow/SU	4	PB	10.00	16.67	23.30	16.67	2.72
weaner/kilo	6	PWN	.52	.90	1.34	.91	.16

Table (11) Distributions of Productivity Levels

	Flow No.	Code	Low	Modal	High	μ	σ
1. Sheep							
wool kilo/SU	12	QW	3.78	4.78	5.78	4.78	0.61
lambs kilos/su	14	QL	7.86	9.83	11.80	9.83	1.19
mutton kilos/su	16	QM	2.95	3.69	4.43	3.69	.45
2. Breeding Cattle							
weaners kilos/su	17	QWNB	4.78	5.98	7.18	5.98	0.73
heifers kilos/su	19	QHF	2.40	3.00	3.60	3.00	.36
cull cows kilos/su	21	QCL	3.18	3.87	4.76	3.97	.48
bull kilos/su	23	QBL	.24	.30	.36	.30	.04
3. Fattening Cattle							
steers kilos/su	25	QST	40.39	50.49	60.59	50.49	6.12
weaners kilos/su	7	QWNF	18.20	22.75	27.30	22.75	2.76

(c) Stock Increases. On page 129 of Appendix I are given the stock increases due to the scheme from the sampled farms at their modal levels. From these base estimates the means and standard deviations of the extra stock carried are derived. Given the total increase as a modal value then the range is assumed to be given by the mode less 20 percent and plus 40 percent (see Table (12)).

Table (12) Total Stock Increases. (s.u.)

	Low	Modal	High
Sheep	13540	16920	23690
Breeding Cattle	10110	12640	17700
Fattening Cattle	8540	10680	14950

The means and standard deviations of the extra stock carried in each year are calculated assuming a triangular distribution, with the standard deviations of the cumulative increases calculated by:-

$$\sigma_{A+B} = (\sigma_A^2 + \sigma_B^2 + 2\,\sigma_A\,\sigma_B)^{\frac{1}{2}} \quad \ldots (58)$$

The total increases are expected to take three years for sheep and five years for cattle. The percentage increases in each year are given in Table (13).

Table (13) Percentage Distribution of Total Stock Increases Over Time

	Percentage of Total Increase				
Year	1	2	3	4	5
Sheep	52	42	6	-	-
Breeding Cattle	34	24	15	14	13
Fattening Cattle	34	24	15	14	13

Given the information in Tables (12) and (13), the relationship in equation (58) and the assumption of a triangular distribution then the means and standard deviation of the stock increases per year and cumulative increases for the total stock numbers can be calculated. These are given in Table (14).

Table (14). Stock Increases. (stock units)

		Code	Year 1	2	3	4	5	6
1. **Increases per Year**								
Sheep	μ	NS[1]	9368	7573	1082			
	σ		1106	990	127			
Breeding	μ	NB[3]	4580	3233	2020	1886	1751	
Cattle	σ		538	380	237	222	206	
Fattening	μ	NF[5]	3869	6600	8307	9900	11378	
Cattle	σ		455	776	977	1164	1338	
2. **Cumulative Increases**								
Sheep	μ	NNS[8]		9368	16941	18023		
	σ			1106	1996	2123		
Breeding	μ	NNB[9]		4580	7813	9833	11719	13470
Cattle	σ			538	918	1155	1377	1583
Fattening	μ	NNF[10]		3869	6600	8307	9900	11379
Cattle	σ			455	776	977	1164	1338

APPENDIX III

THE CORRELATION ANALYSIS

The correlation coefficients are derived by time series analysis. The program and listing of output follows. Data is from national statistics[17] over the years 1958/59 to 1972/73 and the flow numbers given in the output identify the following information:

Flow 1. is the total number of sheep ($*10^7$)

Flow 2. is the total number of beef cattle ($*10^6$)

Flow 6. is the average wool price in cents per kilo on a greasy basis ($*10^1$)

Flow 7. is the wool per sheep in kilos. This was obtained by dividing total wool production by flow 1.

Flow 8. is lamb and mutton production per sheep in kilos and is obtained by dividing total lamb and mutton production by flow 1.

Flow 9. is the production of beef and veal per beast in kilos ($*10^1$) and is obtained by dividing total beef and veal production by flow 2.

Flow 10. is the lamb price in $ per tonne F.O.B. ($*10^2$)

Flow 11. is the mutton price in $ per tonne F.O.B. ($*10^2$)

Flow 12. is beef and veal prices in $ per tonne F.O.B. ($*10^2$)

---

[17] Source: Annual Review of the Sheep Industry 1965/66 and 1973/74, published by New Zealand Meat and Wool Boards' Economic Service.

Flow 13. represents the capital cost of sheep in \$ per head. It is obtained by taking a weighted average of the previous year's approximate revenue from meat and wool. That is flow 6 times flow 7 plus flow 8 times flow 10.

Flow 14. represents the capital cost of beef stock. It is based on the Beef price (flow 12) lagged by one year.

The derived coefficients are then used in the risk programs ANWA and MVSM as shown in Table (16).

Table (16) Correlation Coefficients

Correlation Flow No.	Associated Variables which use the derived coefficient.
1	NS^{1}, NNS^{8}
2	NB^{3}, NF^{5}, NNB^{9}, NNF^{10}.
6	PW^{11}.
7	QW^{12}.
8	QL^{14}, QM^{16}.
9	$QWNF^{7}$, $QWNB^{17}$, QHF^{19}, QCL^{21}, QBL^{23}, QST^{25}.
10	PL^{13}.
11	PM^{15}.
12	PHF^{18}, PCL^{20}, PBL^{22}, PST^{24}.
13	PS^{2},
14	PB^{4}, PWN^{6}.

```
C     PROGRAM CALCULATES CROSS CORRELATION COEFFICIENTS WITH ASSOCIATED
C     T  VALUES AND AUTOCORRELATION COEFFICIENTS (WITH  Z  VALUES) FOR
C     A MAXIMUM NUMBER OF  50  FLOWS AND  20  OBSERVATIONS IN EACH FLOW.
C     THE AUTOCORRELATION IS ASSUMED TO BE NO MORE COMPLEX THAN FIRST
C     ORDER MARKOVIAN.
C     THE DIMENSIONED VARIABLES ARE...
C     A    = INPUT FLOWS CONTAINING UP TO  20  OBSERVATIONS
C     DATO   THE MATRIX CONTAINING ALL THE  A  FLOWS
C     AV     THE AVERAGE OF EACH FLOW  A
C     SM     MATRIX OF DEVIATIONS ABOUT THE MEANS OF THE FLOWS
C     SSM    MATRIX OF SQUARED DEVIATIONS
C     R      MATRIX OF CROSS CORRELATION COEFFICIENTS
C     ALG    ARRAY OF AUTOCORRELATION COEFFICIENTS
C     BATO   THE FIRST DIFFERENCE MATRIX
C     V      ARRAY CONTAINING FLOW NAMES
C     T      THE  T  VALUES ASSOCIATED WITH  R
C     Z      THE  Z  VALUES ASSOCIATED WITH  ALG
C
C     OTHER INPUT VARIABLES
C     IFL    THE NUMBER OF FLOWS
C     NYR    THE NUMBER OF OBSERVATIONS IN EACH FLOW
C
      DIMENSION A(20),DATO(20,30),AV(30),SM(20,30),SSM(20,30),R(30,30),
     1ALG(30),BATO(20,30),V(30),T(30,30),Z(30)
      DO 122 I=1,20
      DO 122 J=1,30
      SM(I,J)=0.
      SSM(I,J)=0.0
  122 DATO(I,J)=0.
      DO 119 I=1,30
      DO 119 J=1,30
      R(I,J)=0.0
      T(I,J)=0.0
  119 CONTINUE
C
C     READ IN DATA
C
      READ(2,102)IFL,NYR
  102 FORMAT(2I4)
      YR=NYR
      IDF=YR-2.
      TYR=SQRT(IDF)
      AYR=1./YR
      IL1=IFL-1
      FL=IFL-1
      AL2=1./(FL-2.)
      AL3=1./(FL-1.)
      AL4=AL3/AL2
      NY=NYR-1
      WRITE(3,140)
  140 FORMAT(/////,30X,' C O R R E L A T I O N    A N A L Y S I S ',////
     1////)
      WRITE(3,129)IFL,NYR
  129 FORMAT(5X,'IN THIS RUN THERE ARE'I5,' FLOWS EACH OF'I5,' OBSERVATI
     1ONS'///5X,' THE FLOWS ARE '//)
C
      READ(2,130)(V(I),I=1,20)
  130 FORMAT(20(A4))
      IF(IFL-20)131,131,132
  132 DO 133 K=21,30,20
      K19=K+19
  133 READ(2,130)(V(I),I=K,K19)
  131 CONTINUE
      WRITE(3,134)(V(I),I=1,IFL)
      WRITE(3,139)
C
      DO 103 J=1,IFL
      READ(2,100)(A(I),I=1,8)
  100 FORMAT(8F10.2)
```

```
      IF(NYR-8)116,116,117
  117 DO 118 K=9,NYR,8
        K7=K+7
  118 READ(2,100)(A(I),I=K,K7)
  116 CONTINUE
C
C     CROSS CORRELATION
C
      DO 103 I=1,NYR
  103 DATO (I,J)=A(I)
C
      WRITE(3,101)((DATO(I,J),I=1,20 ),J=1,IFL)
  101 FORMAT(  5X,10F10.2/  5X,10F10.2//)
C
      DO 104 J=1,IFL
      SUM=0.
      DO 105 I=1,NYR
  105 SUM=SUM+DATO(I,J)
      SUM=SUM/NYR
  104 AV(J)=SUM
C
C
      DO 106 J=1,IFL
      DO 106 I=1,NYR
      TM=DATO(I,J)-AV(J)
      SM(I,J)=TM
  106 SSM(I,J)=TM**2
C
      DO 108 J=1,IFL
      DO 108 I=1,IFL
      TPT=0.
      TPP=0.
      TP =0.
      DO 110 IT=1,NYR
      TP=TP+SM(IT,J)*SM(IT,I)
      TPT=TPT+SSM(IT,I)
  110 TPP=TPP+SSM(IT,J)
      TPP=SQRT(TPP)*SQRT(TPT)
      R(I,J)=TP/TPP
  108 CONTINUE
      WRITE(3,112)
  112 FORMAT('1',30X,' CROSS CORRELATION COEFFICIENTS '///)
      WRITE(3,134)(V(I),I=1,IFL)
  134 FORMAT(  5X,20(1X,A4)/  5X,10(1X,A4)//)
      WRITE(3,139)
  139 FORMAT(/)
C
      WRITE(3,111)((R(I,J),J=1,30 ),I=1,IFL)
  111 FORMAT(  5X,20F5.2/ ,5X,10F5.2//)
      DO 135 I=1,IFL
      DO 135 J=1,IFL
      IF(I-J)141,135,141
  141 T(I,J)=R( I,J)*TYR/ SQRT(1.-R(I,J)**2)
  135 CONTINUE
      WRITE(3,136)IDF
  136 FORMAT('1',30X,' T  VALUES FOR CROSS CORRELATION COEFFICIENTS '/30
     1X,' WITH ' I3,' DEGREES OF FREEDOM '///)
      WRITE(3,111)((T(I,J),J=1,30),I=1,IFL)
C
C     AUTOCORRELATION
C
      DO 114 J=1,IFL
      WT=0.
      WTT=0.
      BATO(1,J)=0.
      DO 113 I=2,NYR
      BATO(I,J)=DATO(I,J)-DATO(I-1,J)
      WT=WT+BATO(I,J)*BATO(I-1,J)
  113 CONTINUE
      DO 127 I=2,NY
      WTT=WTT+BATO(I,J)**2
  127 CONTINUE
  114 ALG(J)=(AL4*WT)/WTT
C
      WRITE(3,115)
  115 FORMAT('1'    /////30X,' THE LAG COEFFICIENTS '///)
      WRITE(3,134)(V(I),I=1,IFL)
```

```
      WRITE(3,139)
      WRITE(3,124)(ALG(J),J=1,IFL)
  124 FORMAT(  5X,20F5.2/ ,5X,10F5.2//)
      DO 137 I=1,IFL
  137 Z(I)=ALG(I)/SQRT(AYR*(1.-ALG(I)**2))
      WRITE(3,138)
  138 FORMAT(///30X,' WITH  Z  VALUES OF '///)
      WRITE(3,124)(Z(I),I=1,IFL)
      CALL EXIT
      END
```

C O R R E L A T I O N   A N A L Y S I S

IN THIS RUN THERE ARE   11 FLOWS EACH OF   15 OBSERVATIONS

THE FLOWS ARE

1   2   6   7   8   9   10   11   12   13   14

4.60 6.05	4.69 5.99	4.71 6.03	4.85 5.89	4.90 6.09	5.02 0.00	5.13 0.00	5.37 0.00	5.73 0.00	6.00 0.00
2.97 4.81	3.02 5.05	3.33 5.28	3.46 5.41	3.56 5.73	3.57 0.00	3.63 0.00	3.85 0.00	4.24 0.00	4.55 0.00
6.64 6.19	8.20 5.64	7.41 5.34	7.21 6.65	7.87 14.40	10.12 0.00	7.74 0.00	7.65 0.00	6.46 0.00	5.05 0.00
5.33 5.49	5.59 5.48	5.67 5.54	5.48 5.47	5.73 5.07	5.56 0.00	5.52 0.00	5.87 0.00	5.62 0.00	5.50 0.00
9.35 9.31	9.55 9.40	9.70 9.35	9.73 9.66	9.45 9.10	9.72 0.00	9.28 0.00	8.73 0.00	9.09 0.00	9.43 0.00
8.01 7.82	7.95 7.78	7.21 7.42	8.27 7.60	8.37 7.84	8.18 0.00	7.60 0.00	7.58 0.00	7.12 0.00	7.58 0.00
3.50 4.20	3.09 4.79	3.25 4.79	2.92 4.70	2.93 5.04	3.15 0.00	3.52 0.00	4.09 0.00	3.70 0.00	3.54 0.00
1.48 2.02	1.32 2.01	1.16 2.73	1.36 2.45	1.32 2.52	1.36 0.00	1.77 0.00	1.87 0.00	1.93 0.00	2.02 0.00
4.43 7.91	4.93 8.40	4.43 9.09	4.57 9.86	4.41 10.60	4.78 0.00	4.70 0.00	4.92 0.00	5.87 0.00	6.25 0.00
6.84 6.12	6.81 7.31	7.53 7.59	7.35 7.44	6.79 8.16	7.28 0.00	8.69 0.00	7.54 0.00	8.06 0.00	6.99 0.00
2.68 6.25	4.43 7.91	4.93 8.40	4.43 9.09	4.57 9.86	4.41 0.00	4.78 0.00	4.70 0.00	4.92 0.00	5.87 0.00

CROSS CORRELATION COEFFICIENTS

1	2	6	7	8	9	10	11	12	13	14									
1.00	0.94	-0.09	-0.31	-0.36	-0.39	0.82	0.89	0.85	0.11	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.94	1.00	0.08	-0.44	-0.23	-0.30	0.90	0.92	0.96	0.18	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
-0.09	0.08	1.00	-0.45	-0.12	0.30	0.09	-0.01	0.16	0.34	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
-0.31	-0.44	-0.45	1.00	-0.10	-0.17	-0.43	-0.40	-0.56	-0.14	-0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
-0.36	-0.23	-0.12	-0.10	1.00	0.33	-0.41	-0.40	-0.14	-0.29	-0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
-0.39	-0.30	0.30	-0.17	0.33	1.00	-0.37	-0.40	-0.23	-0.45	-0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.82	0.90	0.09	-0.43	-0.41	-0.37	1.00	0.90	0.92	0.23	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.89	0.92	-0.01	-0.40	-0.40	-0.40	0.90	1.00	0.89	0.27	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.85	0.96	0.16	-0.56	-0.14	-0.23	0.92	0.89	1.00	0.12	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.11	0.18	0.34	-0.14	-0.29	-0.45	0.23	0.27	0.12	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
0.81	0.95	0.20	-0.46	-0.08	-0.26	0.87	0.85	0.96	0.24	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										

T VALUES FOR CROSS CORRELATION COEFFICIENTS WITH 13 DEGREES OF FREEDOM

```
 0.0010.21-0.31-1.18-1.39-1.53 5.24 7.06 5.80 0.40 4.99 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

10.21 0.00 0.29-1.77-0.86-1.14 7.34 8.4512.23 0.6510.70 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

-0.31 0.29 0.00-1.83-0.44 1.13 0.31-0.03 0.58 1.31 0.75 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

-1.18-1.77-1.83 0.00-0.35-0.61-1.70-1.58-2.46-0.52-1.89 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

-1.39-0.86-0.44-0.35 0.00 1.26-1.62-1.56-0.52-1.07-0.30 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

-1.53-1.14 1.13-0.61 1.26 0.00-1.44-1.58-0.86-1.79-0.96 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

 5.24 7.34 0.31-1.70-1.62-1.44 0.00 7.61 8.29 0.84 6.36 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

 7.06 8.45-0.03-1.58-1.56-1.58 7.61 0.00 7.17 1.03 5.72 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

 5.8012.23 0.58-2.46-0.52-0.86 8.29 7.17 0.00 0.4312.53 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

 0.40 0.65 1.31-0.52-1.07-1.79 0.84 1.03 0.43 0.00 0.89 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

 4.9910.70 0.75-1.89-0.30-0.96 6.36 5.7212.53 0.89 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
```

159

THE LAG COEFFICIENTS

1	2	6	7	8	9	10	11	12	13	14
0.56	0.60	0.19	-0.27	-0.20	-0.21	0.13	-0.18	0.46	-0.26	0.35

WITH Z VALUES OF

1	2	6	7	8	9	10	11	12	13	14
2.59	5.20	0.77	-1.08	-0.81	-0.83	0.52	-0.70	1.99	-1.03	1.46


```
EOF1LINE                      00010001000100 74354 74354 0000500000170 B6500
EOF2F000220002200100000000000000000009330           00
VOL1                           652020601
HDR1LINE                      00010001000100 74354 74354 0000000000000 B6500
HDR2F000200002000100000000000000000009330           00
```

APPENDIX IV

DISAGGREGATION OF THE CASH FLOWS TO OBTAIN THE FIRST DERIVATIVES.

The ANWA computer model requires the analyst to disaggregate the project into its stochastic and deterministic variables so that the flows representing the standard derivations and first derivatives of the stochastic variables can be read into the model. The Tuapeka scheme may be disaggregated as follows: Deterministic variables are denoted by a bar. Stochastic variables have been allocated a code to simplify the equations. These codes have already been introduced in Appendix II. The actual program uses numbers to recognise the flows and these are given in the notation as superscripts above the code for each variable. For example, the extra sheep carried each year is denoted by NS^1. Thus the functional relationships may be written as follows:

(i) Capital Costs =

$$\overline{\text{scheme capital}} + \overline{\text{on-farm capital}} + NS^1 * PS^2 + NB^3 * PB^4 + NF^5 \, (PWN^6 * QWNF^7) \quad \ldots . (59)$$

(ii) Associated Costs =

$$\overline{\text{Scheme associated costs}} + \overline{\text{on-farm associated costs}} + NNS^{8} * \overline{\text{variable cost/s.u.}} + NNB^{9} * \overline{\text{variable cost/s.u.}} + NNF^{10} * \overline{\text{variable cost/s.u.}} \quad \ldots (60)$$

(iii) Benefits =

$$\overline{\text{on farm saved costs and plant salvage}}$$
$$+ NNS^{8} (PW^{11} * QW^{12} + PL^{13} * QL^{14} + PM^{15} * QM^{16})$$
$$+ NNB^{9} (PWN^{6} * QWNB^{17} + PHF^{18} * QHF^{19} + PCL^{20} * QCL^{21} + PBL^{22} * QBL^{23})$$
$$+ NNF^{10} (PST^{24} * QST^{25}) \quad \ldots . (61)$$
$$+ NNS^{8} * PS^{2}$$
$$+ NNB^{9} * PB^{4}$$

Given that the costs and benefits due to the scheme can be written as above in equations 59, 60 and 61, then the expected present value (PV) is

$$E\ (PV) = \sum_{t=0}^{n} \left[\text{Benefits}_t - (\text{Capital Costs}_t + \text{Associated Costs}_t) \right] \alpha^{t} \quad \ldots . (62)$$

with $\alpha = \dfrac{1}{1+i}$

and where the benefits and costs are taken at their expected levels.

The method used to obtain the partial derivatives from the present value equation has been outlined in section 3.1.2 (d). These derivatives (or weights) for the price analysis are as follows:

Table (15) The First Derivatives of the Price Analysis

Flow No.	Code	Derivative
2	$\partial PV / \partial PS$	= −NS + NNS
4	$/ \partial PB$	−NB + NNB
6	$/ \partial PWN$	−NF * QWNF + NNB * QWNG
11	$/ \partial PW$	NNS * QW
13	$/ \partial PL$	NNS * QL
15	$/ \partial PM$	NNS * QM
18	$/ \partial PHR$	NNB * QHF
20	$/ \partial PBL$	NNB * QBL
24	$/ \partial PST$	NNF * QST

The first derivatives for the additional variables may be easily derived in a similar manner.

APPENDIX V

THE ANALYTICAL RISK PROGRAM - ANWA

Listing of program and input data for the full analysis.

```
C       RISK AND UNCERTAINTY PROGRAM  :  A N W A   AN ANALYTICAL
C       RISK EVALUATION PROGRAM FOR WATER RESOURCE PROJECTS.
C
C       THE METHOD USED HAS BEEN DEVELOPED FROM SPRINGER ET AL (1968)
C       AND UTILIZES TAYLOR'S THEOREM FOR CALCULATING THE VARIANCE.
C
C       THE PROGRAM CALCULATES THE EXPECTED PRESENT VALUE AND
C       STANDARD DEVIATIONS FOR CAPITAL PROJECTS.  A GRAPH OF THE
C       CUMMULATIVE DISTRIBUTION OF THE INTERNAL RATE OF RETURN IS
C       ALSO PRESENTED.  SEE ANWA USER MANUAL FOR DETAILS OF INPUT
C       AND OUTPUT.
C
C       THE MAXIMUM NO OF FLOWS ALLOWED IS 30
C       THE MAXIMUM PROJECT LIFE ALLOWED IS 50 YEARS
C
C       THE DIMENSIONED VARIABLES ARE....
C       ACR      THE ARRAY OF AUTOCORRELATION COEFFICIENTS
C       AMTS     THE INPUT FLOWS FOR SD
C       CCR      THE MATRIX OF CROSS CORRELATION COEFFICIENTS
C       ICB    = AN ARRAY TO SIGNIFY WHETHER FLOWS ARE REVENUE (=1)
C                OR COST (=2)
C       IFL      AN ARRAY FOR FLOW NOS
C       JFL      FLOW NOS FOR WGT - MUST= IFL
C       MYR      ARRAY TO SHOW WHICH YEARS THE AMTS FALL
C       NYR      YEARS THAT WGT FALL - MUST= MYR
C       RINT     THE THREE INTEREST RATES
C       WGT      WEIGHTS ASSOCIATED WITH AMTS
C       WGTS     MATRIX  OF WEIGHTS ASSOIATED WITH SD MATRIX
C       SD       THE MATRIX OF STANDARD DEVIATIONS
C       XCCR     THE INPUT FLOWS FOR CCR
C
C       OTHER INPUT VARIABLES ARE....
C       ONAME    THE OPERATORS NAME
C       PNAME    THE PROJECTS NAME
C       IFLT     NO OF FLOWS
C       IWGT     1 IF FLOWS ARE WEIGHTED OTHERWISE ZERO
C       JCCR     FLOW NO OF CROSS CORRELATION COEFFICIENTS
C       JYRS     YEARS OF PROJECT LIFE
C       NACR     NO OF AUTOCORRELATED FLOWS - MUST = IFLT
C       NCCR     THE ORDER OF MATRIX CCR - MUST = IFLT
C       NDATE    DATE DATA INPUTED
C       NOB      NO OF OBSERVATIONS IN EACH FLOW
C       INDEX    THE NUMBER OF INTEREST RATES
C       ISCAL    A SCALE FACTOR FOR INDEX
C       ICNT     A COUNTER FOR INDEX


        DIMENSION JCCR(30)
        DIMENSION
       5          DCT( 50),FLIRR( 50),KFL(30),KYR( 50),DT( 50,30),
       6          AMTS( 50),WGT( 50),IFL(30),MYR( 50),SD( 50,30),
       1ICB(30),CCR(30,30),ACR(30),RINT(3),XCCR(30),ONAME(6),PNAME(6),
       2NDATE(3),JFL(30),NYR( 50),WGTS( 50,30),SDW(40),PCT(40),DCTF(3),
       3PW(40),BC1(40),BC2(40),
       4PRO (40),PRO1(40),ITABL(310)
C
C       READ IN AND WRITE OPERATORS NAME, PROJECTS NAME AND THE DATE
C
        READ(2,750)ONAME,PNAME,NDATE
  750   FORMAT(2(6A4),3I2)
        WRITE(3,751)PNAME,ONAME,NDATE
  751   FORMAT('1',////,45X,6A4,//80X,6A4,2X,I2'/'I2'/'I2)
        READ(2,10)(ITABL(I),I=1,310)
   10   FORMAT(15(20I4/),10I4)
C
C       READ IN THE NO OF FLOWS, PROJECT LIFE, WEIGHTING CODE AND
C       INTEREST RATES
C
        READ(2,716) IFLT,JYRS,IWGT,RINT,INDEX,ISCAL
  716   FORMAT(I2,I3,I2,3F4.2,I3,I1)
        WRITE(3,748)IFLT,JYRS,RINT
  748   FORMAT(      /////'  THE NO OF FLOWS, PROJECT LIFE AND THREE INTER
       1T RATES ARE '///2X,I2,5X,I3,5X,3F6.2/////)
C
        A1=RINT(1)*100.0-99.5
```

```
      A2=RINT(2)*100.0-99.5
      A3=RINT(3)*100.0-99.5
      I1=A1
      I2=A2
      I3=A3
C
C
      JYRS=JYRS +1
      DO 760 I=1,3
      DO 760 J=1,100
  760 DT(J,I)=0.0
C
C     READ IN THE BENEFIT FLOW, THE ASSOCIATED COST FLOW AND THE
C     CAPITAL COST FLOW AND PRINT AS INPUTED
C
      DO 603 I=1,3
      READ(2,100)KFL(I),NOBK,(KYR(J),DCT(J),J=1,6)
  100 FORMAT(3X,2I2,6(I3,F8.0))
      IF(NOBK-6)600,600,601
  601 DO 602 K = 7,NOBK,6
      K5 = K +5
      READ(2,102)(KYR(J),DCT(J),J=K,K5)
  102 FORMAT(7X,6(I3,F8.0))
  602 CONTINUE
  600 KYR(NOBK+1) = KYR(NOBK) + 1
C
C     INTERPOLATE
C
      LL=1
      DO 617 J=1,NOBK
      K= KYR(J+1)-KYR(J)
      DT(LL,I)=DCT(J)
      IF(K-1)617,617,619
  619 ADD=(DCT(J+1)-DCT(J))/K
      L=LL+K-2
      DO 620 M=LL,L
  620 DT(M+1,I)=DT(M,I)+ADD
      LL=L+1
  617 LL=LL+1
      DO 621 J=LL, 50
  621 DT(J,I)=DCT(NOBK)
  603 CONTINUE
C
C     PRINT IN CONSECUTIVE ORDER THE FULL FLOWS OF BENEFITS,
C     ASSOCIATED COSTS AND CAPITAL COSTS
C
  775 WRITE(3,604)
  604 FORMAT(///////' INPUT FLOWS TO BE DISCOUNTED'////)
      WRITE(3,605)(DT(I,1),I=1,JYRS)
  605 FORMAT(////' BENEFITS'6X, 6(10F10.0/15X)//)
      WRITE(3,623)(DT(I,2),I=1,JYRS)
  623  FORMAT(//' ASSOC COSTS'3X, 6(10F10.0/15X)//)
      WRITE(3,624)(DT(I,3),I=1,JYRS)
  624 FORMAT(//' CAPITAL COSTS'1X, 6(10F10.0/15X))
C
C     CALCULATE THE NET CASH FLOW
C
  763 DO 607 I =1,JYRS
      FLIRR(I) =DT(I,1)-DT(I,2)-DT(I,3)
  607 CONTINUE
C
C     CALCULATE THE EXPECTED IRR
C
      CALL ALIRR(FLIRR,JYRS,0.0001,10,RRT)
      RR =RRT*100.0
C
C
C     CALCULATE THE PRESENT VALUE OF THE CASH FLOWS
C     SEPERATELY AT (INDEX/ISCAL) INTEREST RATES
C
      ICNT=0
      RIN= 1.0
      DO 728 I=1,INDEX,ISCAL
      ICNT=ICNT+1
      RIN=RIN+0.01*ISCAL
      DO 608 J=1,3
      DCTF(J)=0.0
```

```
      DCTF(J) =DCTF(J)+DT(1,J)
      DO 608 IT =2,JYRS
      ITN = IT-1
      DCTF(J)= DCTF(J)+ DT(IT,J)/RIN**ITN
  608 CONTINUE
      PW(I)=DCTF(1)-DCTF(2) - DCTF(3)
  773 PVTP=PW(I)


  774 BC1(I)= (DCTF(1)-DCTF(2))/DCTF(3)
      BC2(I)=DCTF(1)/(DCTF(2) +DCTF(3))
  728 CONTINUE
      DO 74 I=1,30
      DO 74 J=1,50
      WGTS(J,I)=1.0
   74 SD(J,I)=0.0
      IF(IWGT)107,107,106
C
C     READ IN WEIGHTS CORRESPONDING TO SD DATA AND PRINT
C
  106 CONTINUE
      DO 1 I =1, IFLT
      READ(2,100)JFL(I),NOBS,(NYR(J),WGT(J),J=1,6)
      IF(NOBS-6)2,2,3
    3 DO 4 K=7,NOBS,6
      K5 =K+5
      READ(2,102)(NYR(J),WGT(J),J=K,K5)
    4 CONTINUE
    2 NYR(NOBS+1)=NYR(NOBS)+1
C
C     INTERPOLATE
C
      LL=1
      DO 5 J=1,NOBS
      K= NYR(J+1)-NYR(J)
      WGTS(LL,I)= WGT(J)
      IF(K-1)5,5,6
    6 ADD=(WGT(J+1)-WGT(J))/K
      L=LL +K-2
      DO 7 M=LL,L
    7 WGTS(M+1,I)= WGTS(M,I)+ADD
      LL =L +1
    5 LL=LL+1
C
      DO 8 J=LL,50
    8 WGTS(J,I)=WGT(NOBS)
    1 CONTINUE
C
C     PRINT FULL MARIX OF WEIGHTS
C
  764 WRITE(3,103)
  103 FORMAT('1',////30X,' MATRIX OF WEIGHTS CORRESPONDING'/35X,' TO
     1 DATA VALUES'///6X' FLOW NO'//)
      DO 9 J=1,IFLT
      WRITE(3,104)JFL(J),(WGTS(I,J),I=1,JYRS)
  104 FORMAT(/8X,I2,5X,12(9F11.2,/15X))
    9 CONTINUE
  107 CONTINUE
C
C     READ IN THE STANDARD DEVIATION DATA AND PRINT
C
      DO 78 I=1,IFLT
      READ(2,700)ICB(I),IFL(I),NOB,(MYR(J),AMTS(J),J=1,6)
  700 FORMAT(2X,I1,2I2,6(I3,F6.0))
      IF(NOB-6)70,70,71
   71 DO 72 K=7,NOB,6
      K5=K+5
      READ(2,701)(MYR(J),AMTS(J),J=K,K5)
  701 FORMAT(7X,6(I3,F8.0))
   72 CONTINUE
   70 MYR(NOB+1)=MYR(NOB)+1
C
C      INTERPOLATE FOR YEARS NOT FILLED IN
C
      LL=1
      DO 73 J=1,NOB
      K=MYR(J+1)-MYR(J)
```

```
      SD(LL,I)=AMTS(J)
      IF(K-1)73,73,75
   75 ADD=(AMTS(J+1)-AMTS(J))/K
      L=LL+K-2
      DO 76 M=LL,L
   76 SD(M+1,I)=SD(M,I)+ADD
      LL=L+1
   73 LL=LL+1
      DO 77 J=LL,50
   77 SD(J,I)=AMTS(NOB)
   78 CONTINUE
C
C     PRINT FULL SD FLOW
C
  765 WRITE(3,744)
  744 FORMAT('1',/////35X,' THE MATRIX OF STANDARD DEVIATIONS '///1X,'.
     1ENT, FLOW NO '//)
      DO 747 J=1,IFLT
      WRITE(3,733) ICB(J),IFL(J),(SD(I,J),I=1,JYRS)
  733 FORMAT(/2X,I1,5X,I2,5X,12(9F11.2,/15X))
  747 CONTINUE
C
  777 READ(2,776)NCCR
  776 FORMAT(2X,I2)
      IF(NCCR-90)766,800,800
  766 DO 717 I=1,30
      DO 717 J=1,30
  717 CCR(I,J)=0.0
C
C     READ CROSS CORRELATION INFO AND PRINT
C
      DO 719 J=1,IFLT
      READ(2,718)JCCR(J),NCCR,(XCCR(N),N =1,14)
  718 FORMAT(2I2,14F5.2)
      IF(NCCR-14)729,729,720
  720 DO 721 K=15,NCCR,14
      K13=K+13
  721 READ(2,722)(XCCR(M),M=K,K13)
  722 FORMAT(4X,14F5.2)
  729 DO 719 I=1,NCCR
      CCR(I,J)=XCCR(I)
  719 CONTINUE
      JFLT=IFLT-1
      DO 761 I=1,JFLT
      K=I+1
      DO 761 J=K,IFLT
  761 CCR(J,I)=CCR(I,J)
  767 WRITE(3,745)
  745 FORMAT('1',/////35X,' THE MATRIX OF CROSS CURRELATION COFFICIENT
     1'/////)
      DO 762 J=1,IFLT
      WRITE(3,731)JCCR(J),(CCR(I,J),I=1,30)
  731 FORMAT(/8X,I2,5X,20F5.2/15X,10F5.2/)
  762 CONTINUE
C
  768 DO 742 I=1,30
  742 ACR(I)=0.0
C
C     READ AUTOCORRELATION INFO AND PRINT
C
      READ(2,723)NACR,(ACR(I),I=1,14)
  723 FORMAT(2X,I2,14F5.2)
      IF(NACR-14)724,724,725
  725 DO 726 K=15,NACR,14
      K13=K+13
  726 READ(2,727)(ACR(I),I=K,K13)
  727 FORMAT(4X,14F5.2)
  724 CONTINUE
  769 WRITE(3,746)
  746 FORMAT(/////////35X,' THE AUTOCORRELATION COEFFICIENTS '///)
      WRITE(3,732)(ACR(I),I=1,30)
  732 FORMAT(15X,20F5.2//15X,10F5.2)
C
      RIN=1.
      DO 752 M=1,INDEX,ISCAL
      RIN= RIN + 0.01*ISCAL
      VP=0.0
```

```
C
C     COMPUTE VARIANCE
C
      DO 703 J=1,IFLT
      VP = VP +SD(1,J)**2 * WGTS(1,J)**2
      DO 703 IT=2,JYRS
      ITM=2*(IT-1)
      VP = VP+(SD(IT,J)**2 /RIN**ITM) * WGTS(IT,J)**2
  703 CONTINUE
C
C     COMPUTE COVARIANCE
C
      DO 704 I=1,IFLT
      JJ=I+1
      DO 704 J=JJ,IFLT
C
C
C     ADD COVARIANCE OF LIKE FLOWS
C     SUBTRACT COVARIANCE OF UNLIKE FLOWS
C
  705 CC=0.0
      CC = CC +CCR(I,J)*SD(1,I)*SD(1,J)*WGTS(1,I) *WGTS(1,J)
      DO 706 IT=2,JYRS
      ITM=2*(IT-1)
      CC = CC+(CCR(I,J) *SD(IT,I)*SD(IT,J)/RIN**ITM) * WGTS(IT,I)*
     1WGTS(IT,J)
  706 CONTINUE
      VP=VP+CC*2
  704 CONTINUE
C
C     COMPUTE AUTOCOVARIANCE
C
      AC=0.0
      DO 711 J=1,IFLT
      ACZ=0.0
      ACZ=ACZ +(2*ACR(J)*SD(1,J)*SD(2,J)/RIN)      *WGTS(1,J)*WGTS(2,J)
      DO 802 K=2,JYRS
      LL=K+1
      DO 802 L=LL,JYRS

  805 CONTINUE
  712 ITM=K+L-2
      XPONT=ACR(J)**IABS(K-L)
  802 ACZ= ACZ+(XPONT              *SD(K,J)*SD(L,J)/RIN**ITM) *WGTS(K,J)*
     1WGTS(L,J)
      IF(ICB(J)-1)779,779,801
  779 AC=AC+ACZ*2
      GO TO 711
  801 AC=AC-ACZ*2
  711 CONTINUE
      VP=VP+AC



C
  772 SDW(M)=SQRT(ABS(VP))
      PCT(M) = SDW(M) * 1.96
  752 CONTINUE
      PT1A = PW(I1)-PCT(I1)
      PT1B = PW(I1)+PCT(I1)
      PT2A = PW(I2)-PCT(I2)
      PT2B = PW(I2)+PCT(I2)
      PT3A = PW(I3)-PCT(I3)
      PT3B = PW(I3)+PCT(I3)
C
      WRITE(3,715)
  715 FORMAT('1',14X,' RESULTS ',/////)
C
C     PRINT INTEREST RATE,EXPECTED PRESENT VALUE,BENEFIT COST RATIOS
C     STANDARD DEVIATION AND 95 PERCENTILE LIMITS
C
      WRITE(3,714)I1,I2,I3
  714 FORMAT(21X,' INTEREST RATE'11X,3I15///)
      WRITE(3,610)PW(I1),PW(I2),PW(I3)
  610 FORMAT(9X,'EXPECTED  -  PRESENT VALUE',14X,3F15.2//)
      WRITE(3,626) SDW(I1),SDW(I2),SDW(I3)
  626 FORMAT(21X,' STANDARD DEVIATION'9X,3F15.2//)
```

```
      WRITE(3,618)PT1A,PT2A,PT3A,PT1B,PT2B,PT3B
  618 FORMAT(21X,' 95 PERCENTILE LIMITS- FROM'1X,3F15.2/42X'- TO'3X,3F
     1.2//)
C
      WRITE(3,614)RR
  614 FORMAT(/////21X,' INTERNAL RATE OF RETURN'5X,F15.2/////)
C
      WRITE(3,615)BC1(I1),BC1(I2),BC1(I3)
  615 FORMAT(
     121X,' BENEFIT COST RATIOS'/21X' 1) BENEFITS NET OF ASSOCIATED'/
     221X,'    COSTS TO CAPITAL COSTS'4X,3F15.2/)
      WRITE(3,622)BC2(I1),BC2(I2),BC2(I3)
  622 FORMAT(
     321X,' 2) BENEFITS TO TOTAL COSTS'2X,3F15.2)
C


      CALL ABRGH(PW,SDW,ITABL,PRO ,ICNT )
      CALL ABPGZ(ITABL,PW(I1),SDW(I1),A)
      CALL ABPGZ(ITABL,PW(I2),SDW(I2),B)
      CALL ABPGZ(ITABL,PW(I3),SDW(I3),C)
      WRITE(3,625)A,B,C
  625 FORMAT(/////21X,' PER CENT PROB OF PV > ZERO'3F15.2//)
      DO 758 I=1,ICNT
  758 PRO1(I)=PRO(I)
      CALL ABGPH(PRO ,PRO1 ,ICNT ,2.5,2.5)
      GO TO 777
  800 CALL EXIT
C
      END
```

TUAPEKA FULL ANALYSIS

B A BELL 26/11/74

THE NO OF FLOWS, PROJECT LIFE AND THREE INTEREST RATES ARE

25 36 1.09 1.10 1.11

INPUT FLOWS TO BE DISCOUNTED

BENEFITS	0.	16387.	301221.	503736.	581699.	656605.	724625.	724625.	724625.	724625.
	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.
	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.	724625.
	724625.	724625.	724625.	724625.	724625.	1066215.	0.			
ASSOC COSTS	0.	23203.	104494.	161629.	184955.	201780.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	187674.	0.			
CAPITAL COSTS	377744.	253680.	275763.	216517.	239797.	264714.	235525.	235525.	235525.	235525.
	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.
	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.	235525.
	235525.	235525.	235525.	235525.	235525.	0.	0.			

MATRIX OF WEIGHTS CORRESPONDING
TO SD DATA VALUES

FLOW NO									
1	0.00	-6.65	-6.65	-6.65	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00								
2	0.00	-9368.00	-7573.00	-1062.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18023.00
	0.00								
3	0.00	-16.67	-16.67	-16.67	-16.67	-16.67	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00								
4	0.00	-4580.00	-3233.00	-2020.00	-1886.00	-1751.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13470.00
	0.00								
5	0.00	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70
	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70
	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70
	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70	-20.70
	0.00								
6	0.00	-88020.00	-122762.00	-142263.00	-166424.00	-186770.00	-178298.00	-178298.00	-178298.00
	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00
	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00
	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	-178298.00	80551.00
	0.00								
7	0.00	0.00	-6006.00	-7559.00	-9009.00	-1255.00	-1255.00	-1255.00	-1255.00
	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00
	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00
	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00	-1255.00
	0.00								
8	0.00	0.00	6.40	6.40	6.40	6.40	6.40	6.40	6.40
	6.40	6.40	6.40	6.40	6.40	6.40	6.40	6.40	6.40
	6.40	6.40	6.40	6.40	6.40	6.40	6.40	6.40	6.40
	6.40	6.40	6.40	6.40	6.40	6.40	6.40	6.40	13.05
	0.00								
9	0.00	0.00	6.63	6.63	6.63	6.63	6.63	6.63	6.63
	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63
	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63
	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	23.30
	0.00								
10	0.00	0.00	28.66	28.66	28.66	28.66	28.66	28.66	28.66
	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66
	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66
	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66	28.66
	0.00								
11	0.00	0.00	44779.00	80978.00	86150.00	86150.00	86150.00	86150.00	86150.00
	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00
	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00
	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00	86150.00

	0.00								
12	0.00	0.00	5808.00	10503.00	11174.00	11174.00	11174.00	11174.00	11174.00
	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00
	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00
	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00	11174.00
	0.00								
13	0.00	0.00	92087.00	166530.00	177166.00	177166.00	177166.00	177166.00	177166.00
	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00
	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00
	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00	177166.00
	0.00								
14	0.00	0.00	5621.00	10165.00	10814.00	10814.00	10814.00	10814.00	10814.00
	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00
	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00
	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00	10814.00
	0.00								
15	0.00	0.00	34569.00	62512.00	66505.00	66505.00	66505.00	66505.00	66505.00
	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00
	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00
	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00	66505.00
	0.00								
16	0.00	0.00	2810.00	5082.00	5407.00	5407.00	5407.00	5407.00	5407.00
	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00
	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00
	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00	5407.00
	0.00								
17	0.00	0.00	4168.00	7110.00	8948.00	10664.00	12258.00	12258.00	12258.00
	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00
	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00
	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00	12258.00
	0.00								
18	0.00	0.00	13740.00	23439.00	29499.00	35157.00	40410.00	40410.00	40410.00
	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00
	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00
	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00	40410.00
	0.00								
19	0.00	0.00	3601.00	6485.00	8161.00	9727.00	11180.00	11180.00	11180.00
	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00
	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00
	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00	11180.00
	0.00								
20	0.00	0.00	18183.00	31018.00	39037.00	46524.00	53476.00	53476.00	53476.00
	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00
	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00
	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00	53476.00
	0.00								
21	0.00	0.00	2840.00	4844.00	6069.00	7266.00	8351.00	8351.00	8351.00
	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00
	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00
	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00	8351.00
	0.00								
22	0.00	0.00	1374.00	2344.00	2950.00	3516.00	4041.00	4041.00	4041.00
	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00
	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00
	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00	4041.00
	0.00								
23	0.00	0.00	3389.00	5782.00	7270.00	8672.00	9968.00	9968.00	9968.00
	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00
	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00
	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00	9968.00
	0.00								
24	0.00	0.00	195346.00	333234.00	419420.00	499851.00	574526.00	574526.00	574526.00
	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00
	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00

	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00	574526.00
	0.00								
25	0.00	0.00	2631.00	4488.00	4649.00	6732.00	7738.00	7738.00	7738.00
	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00
	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00
	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00	7738.00
	0.00								

THE MATRIX OF STANDARD DEVIATIONS

IDENT,	FLOW NO									
2	1	0.00	1106.00	890.00	127.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00								
2	2	0.00	1.25	1.25	1.25	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.25
		0.00								
2	3	0.00	538.00	380.00	237.00	222.00	206.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00								
2	4	0.00	16.67	16.67	16.67	16.67	16.67	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.67
		0.00								
2	5	0.00	455.00	776.00	977.00	1164.00	1338.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	0.00
		0.00								
2	6	0.00	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
		0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
		0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
		0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
		0.00								
2	7	0.00	2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76
		2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76
		2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76
		2.76	2.76	2.76	2.76	2.76	2.76	2.76	2.76	0.00
		0.00								
1	8	0.00	0.00	1106.00	1996.00	2123.00	2123.00	2123.00	2123.00	2123.00
		2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00
		2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00
		2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00	2123.00
		0.00								
1	9	0.00	0.00	538.00	918.00	1155.00	1377.00	1583.00	1583.00	1583.00
		1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00
		1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00
		1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00	1583.00
		0.00								
1	10	0.00	0.00	455.00	776.00	977.00	1164.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00
		1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00	1338.00
		0.00								
1	11	0.00	0.00	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.00								

1	12	0.00	0.00	0.61	0.61	0.61	0.61	0.61	0.61	0.61
		0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
		0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
		0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
		0.61								
1	13	0.00	0.00	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.00								
1	14	0.00	0.00	1.19	1.19	1.19	1.19	1.19	1.19	1.19
		1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
		1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
		1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
		0.00								
1	15	0.00	0.00	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		0.00								
1	16	0.00	0.00	0.45	0.45	0.45	0.45	0.45	0.45	0.45
		0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
		0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
		0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
		0.00								
1	17	0.00	0.00	0.73	0.73	0.73	0.73	0.73	0.73	0.73
		0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73
		0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73
		0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73
		0.00								
1	18	0.00	0.00	0.18	0.18	0.18	0.18	0.18	0.18	0.18
		0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
		0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
		0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
		0.00								
1	19	0.00	0.00	0.36	0.36	0.36	0.36	0.36	0.36	0.36
		0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
		0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
		0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
		0.00								
1	20	0.00	0.00	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.00								
1	21	0.00	0.00	0.48	0.48	0.48	0.48	0.48	0.48	0.48
		0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
		0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
		0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
		0.00								
1	22	0.00	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		0.00								
1	23	0.00	0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		0.00								
1	24	0.00	0.00	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

		0.00								
1	25	0.00	0.00	6.12	6.12	6.12	6.12	6.12	6.12	6.12
		6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12
		6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12
		6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12	6.12
		0.00								

THE MATRIX OF CROSS CORRELATION COFFICIENTS

1	1.00	0.11	0.94	0.81	0.94	0.81	-0.39	1.00	0.94	0.94	-0.09	-0.31	0.82	-0.36	0.89	-0.36	-0.39	0.85	-0.39	0.85	-0.39	0.85	-0.39	0.85	-0.39	0.00	0.00	0.00	0.00	0.00
2	0.11	1.00	0.18	0.24	0.18	0.24	-0.45	0.11	0.18	0.18	0.34	-0.14	0.23	-0.29	0.27	-0.29	-0.45	0.12	-0.45	0.12	-0.45	0.12	-0.45	0.12	-0.45	0.00	0.00	0.00	0.00	0.00
3	0.94	0.18	1.00	0.95	1.00	0.95	-0.30	0.94	1.00	1.00	0.08	-0.44	0.90	-0.23	0.92	-0.23	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.00	0.00	0.00	0.00	0.00
4	0.81	0.24	0.95	1.00	0.95	1.00	-0.26	0.81	0.95	0.95	0.20	-0.46	0.87	-0.08	0.85	-0.08	-0.26	0.96	-0.26	0.96	-0.26	0.96	-0.26	0.96	-0.26	0.00	0.00	0.00	0.00	0.00
5	0.94	0.18	1.00	0.95	1.00	0.95	-0.30	0.94	1.00	1.00	0.08	-0.44	0.90	-0.23	0.92	-0.23	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.00	0.00	0.00	0.00	0.00
6	0.81	0.24	0.95	1.00	0.95	1.00	-0.26	0.81	0.95	0.95	0.20	-0.46	0.87	-0.08	0.85	-0.12	-0.26	0.96	-0.26	0.96	-0.26	0.96	-0.26	0.96	-0.26	0.00	0.00	0.00	0.00	0.00
7	-0.39	-0.45	-0.30	-0.26	-0.30	-0.26	1.00	-0.39	-0.30	-0.30	0.30	-0.17	-0.37	0.33	-0.40	0.33	1.00	-0.23	1.00	-0.23	1.00	-0.23	1.00	-0.23	1.00	0.00	0.00	0.00	0.00	0.00
8	1.00	0.11	0.94	0.81	0.94	0.81	-0.39	1.00	0.94	0.94	-0.09	-0.31	0.82	-0.36	0.89	-0.36	-0.39	0.85	-0.39	0.85	-0.39	0.85	-0.39	0.85	-0.39	0.00	0.00	0.00	0.00	0.00
9	0.94	0.18	1.00	0.95	1.00	0.95	-0.30	0.94	1.00	1.00	0.08	-0.44	0.90	-0.23	0.92	-0.23	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.00	0.00	0.00	0.00	0.00
10	0.94	0.18	1.00	0.95	1.00	0.95	-0.30	0.94	1.00	1.00	0.08	-0.44	0.90	-0.23	0.92	-0.23	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.96	-0.30	0.00	0.00	0.00	0.00	0.00
11	-0.09	0.34	0.08	0.20	0.08	0.20	0.30	-0.09	0.08	0.08	1.00	-0.45	0.09	-0.12	-0.01	-0.12	0.30	0.16	0.30	0.16	0.30	0.16	0.30	0.16	0.30	0.00	0.00	0.00	0.00	0.00
12	-0.31	-0.14	-0.44	-0.46	-0.44	-0.46	-0.17	-0.31	-0.44	-0.44	-0.45	1.00	-0.43	-0.10	-0.40	-0.10	-0.17	-0.56	-0.17	-0.56	-0.17	-0.56	-0.17	-0.56	-0.17	0.00	0.00	0.00	0.00	0.00
13	0.82	0.23	0.90	0.87	0.90	0.87	-0.37	0.82	0.90	0.90	0.09	-0.43	1.00	-0.41	0.90	-0.41	-0.37	0.92	-0.37	0.92	-0.37	0.92	-0.37	0.92	-0.37	0.00	0.00	0.00	0.00	0.00
14	-0.36	-0.29	-0.23	-0.08	-0.23	-0.08	0.33	-0.36	-0.23	-0.23	-0.12	-0.10	-0.41	1.00	-0.40	1.00	0.33	-0.14	0.33	-0.14	0.33	-0.14	0.33	-0.14	0.33	0.00	0.00	0.00	0.00	0.00
15	0.89	0.27	0.92	0.85	0.92	0.85	-0.40	0.89	0.92	0.92	-0.01	-0.40	0.90	-0.40	1.00	-0.40	-0.40	0.89	-0.40	0.89	-0.40	0.89	-0.40	0.89	-0.40	0.00	0.00	0.00	0.00	0.00
16	-0.36	-0.29	-0.23	-0.08	-0.23	-0.12	0.33	-0.36	-0.23	-0.23	-0.12	-0.10	-0.41	1.00	-0.40	1.00	0.33	-0.14	0.33	-0.14	0.33	-0.14	0.33	-0.14	0.33	0.00	0.00	0.00	0.00	0.00
17	-0.39	-0.45	-0.30	-0.26	-0.30	-0.26	1.00	-0.39	-0.30	-0.30	0.30	-0.17	-0.37	0.33	-0.40	0.33	1.00	-0.23	1.00	-0.23										

176.

```
       1.00-0.23 1.00-0.23 1.00 0.00 0.00 0.00 0.00 0.00

18      0.85 0.12 0.96 0.96 0.96 0.96-0.23 0.85 0.96 0.96 0.16-0.56 0.92-0.14 0.89-0.14-0.23 1.00-0.23 1.00
       -0.23 1.00-0.23 1.00-0.23 0.00 0.00 0.00 0.00 0.00

19     -0.39-0.45-0.30-0.26-0.30-0.26 1.00-0.39-0.30-0.30 0.30-0.17-0.37 0.33-0.40 0.33 1.00-0.23 1.00-0.23
        1.00-0.23 1.00-0.23 1.00 0.00 0.00 0.00 0.00 0.00

20      0.85 0.12 0.96 0.96 0.96 0.96-0.23 0.85 0.96 0.96 0.16-0.56 0.92-0.14 0.89-0.14-0.23 1.00-0.23 1.00
       -0.23 1.00-0.23 1.00-0.23 0.00 0.00 0.00 0.00 0.00

21     -0.39-0.45-0.30-0.26-0.30-0.26 1.00-0.39-0.30-0.30 0.30-0.17-0.37 0.33-0.40 0.33 1.00-0.23 1.00-0.23
        1.00-0.23 1.00-0.23 1.00 0.00 0.00 0.00 0.00 0.00

22      0.85 0.12 0.96 0.96 0.96 0.96-0.23 0.85 0.96 0.96 0.16-0.56 0.92-0.14 0.89-0.14-0.23 1.00-0.23 1.00
       -0.23 1.00-0.23 1.00-0.23 0.00 0.00 0.00 0.00 0.00

23     -0.39-0.45-0.30-0.26-0.30-0.26 1.00-0.39-0.30-0.30 0.30-0.17-0.37 0.33-0.40 0.33 1.00-0.23 1.00-0.23
        1.00-0.23 1.00-0.23 1.00 0.00 0.00 0.00 0.00 0.00

24      0.85 0.12 0.96 0.96 0.96 0.96-0.23 0.85 0.96 0.96 0.16-0.56 0.92-0.14 0.89-0.14-0.23 1.00-0.23 1.00
       -0.23 1.00-0.23 1.00-0.23 0.00 0.00 0.00 0.00 0.00

25     -0.39-0.45-0.30-0.26-0.30-0.26 1.00-0.39-0.30-0.30 0.30-0.17-0.37 0.33-0.40 0.33 1.00-0.23 1.00-0.23
        1.00-0.23 1.00-0.23 1.00 0.00 0.00 0.00 0.00 0.00
```

THE AUTOCORRELATION COEFFICIENTS

```
        0.56-0.26 0.80 0.35 0.80 0.35-0.21 0.56 0.80 0.80 0.19-0.27 0.13-0.20-0.18-0.20-0.21 0.46-0.21 0.46
       -0.21 0.46-0.21 0.46-0.21 0.00 0.00 0.00 0.00 0.00
```

APPENDIX VI

ANWA - LAYOUT FOR CARD INPUT

Card Type

(i) Read in operator's name (ONAME), the project's name (PNAME) and the date (NDATE).

ONAME	PNAME	NDATE
1 – 24	25 – 48	49 – 54

(ii) Read in the No. of uncertain cash flows (IFLT), project life (JYRS), weighting code (IWGT), (IWGT - 1 for the weighted flows, IWGT = 0 for unweighted flows), interest rates (RINT) any three rates between 1 and 40, 1.09 = 9%. Finally INDEX = 40 and ISCAL = 1.

IFLT	JYRS	IWGT		RINT		INDEX	ISCAL
1 2	3 5	6 7	8 11	12 15	16 19	20 22	23

(maximum IFLT = 30, JYRS = 50)

(iii) Read in the Benefit Flow (1), Associated Cost Flow (2) and Capital Cost Flow (3) at their expected levels, in the above order.

KFL	NOBK	KYR	DCT		KYR	DCT
45	6 7	8 10	11 18	- - - - - - - - -		75
			1	to	6	

where KFL = flow No. 1,2, or 3

NOBK = No. of observations in the flow

KYR = period particular flow occurs in

DCT = value of flow

(iv) Read in the weights (if any) that correspond to the uncertain cash flows.

JFL	NOBS	NYR	WGT		NYR	WGT
4 5	6 7	8 10	11 18	- - - - - -		75
			1	to		6

where JFL = Flow No.

NOBS = No. of observations in the flow

NYR = period observation falls

WGT = weighting associated with an uncertain cash flow.

(v) Read in the standard deviations associated with the uncertain cash flow.

ICB	IFL	NOB	MYR	AMTS		MYR	AMTS
1	4 5	6 7	8 10	11 18	- - - - - -		75
				1	to		6

where ICB = 1 for a Benefit flow

= 2 for a Cost flow

IFL = flow No., IFL = JFL for the SD associated with the weight for the uncertain cash flow.

NOB = No. of observations in the flow

MYR = period observation falls

AMTS = value of observations

(vi) Read the Criss Correlations Coefficients. (Note – only the lower triangular matrix must be specified).

JCCR NCCR XCCR

1 2 3 4 5 9 10 14

1 to 14

where JCCR = flow No.

NCCR = No. of observations

XCCR = value of observations

(vii) Read the Auto-correlations Coefficients

NACR ACR

3 4 5 9

1 to 14

where NACR = No. of observations

ACR = value of coefficients

APPENDIX VII

MVSM - LISTING OF PROGRAM, INPUT DATA AND OUTPUT

```
C
C     A PROGRAM FOR GENERATING VARIABLES FROM MULTIVARIATE NORMAL
C     DISTRIBUTIONS BY MONTE CARLO SIMULATION. INCORPERAED ARE CROSS
C     AND AUTOCORRELATION TO CALCULATE THE MEAN AND STANDARD DEVIATION
C     OF CAPITAL PROJECTS.
C
C     THE DIMENSIONED VARIABLES ARE ---
C
C     AVPQ   =   VECTOR OF PRICES
C     C          LOWER TRIANGULAR SQUARE ROOT MATRIX OF R
C     CFL        NET CASH FLOW FOR PROJECT
C     DCT        VALUE OF FLOW IN DT
C     DT         MATRIX CONTAINING DETERMINISTIC CASH FLOWS WHERE
C                NOT ASSOCIATED WITH PRICE VARIABLES
C     FLIRR      THE NET CASH FLOW OF DETERMINISTIC VARIABLES
C     GENPQ      THE GENERATED RANDOM NORMAL PRICES
C     ITABL      TABLE OF RANDOM NORMAL NUMBERS 0000 TO 4990
C     JFL        FLOW NUMBERS OF WEIGHTS
C     KFL        FLOW NUMBER OF DETERMINISTIC CASH FLOW
C     KYR        PERIOD VALUE IN DCT FALLS
C     NYR        PERIOD WEIGHTS FALL IN
C     PV         VECTOR OF PRESENT VALUES
C     R          VARIANCE COVARIANCE MATRIX
C     RBEYR      AUTOCORRELATION COEFFICIENTS ASSOCIATED WITH AVPQ
C     RINYR      CROSS CORRELATION COEFFICIENTS ASSOCIATED WITH AVPQ
C     RR         VECTOR OF IRRS
C     SDPQ       VECTOR OF STANDARD DEVIATIONS ASSOCIATED WITH AVPQ
C     WGT        VALUE OF WEIGHTS ASSOCIATED WITH PRICE VARIABLES
C     WGTS       MATRIX OF WEIGHTS OVER ALL VARIABLES AND YEARS
C     Z          VECTOR OF STANDARDISED RANDOM NORMAL NUMBERS
C
C     OTHER INPUT VARIABLES
C
C     NO         NUMBER OF SIMULATIONS
C     NOB        NUMBER OF OBSERVATIONS OF RINYR AND RBEYR
C     NOBK       NUMBER OF OBSERVATIONS OF DCT
C     NOBS       NUMBER OF OBSERVATIONS OF WGT
C     NPQ        NUMBER OF STOCHASTIC PRICE VARIABLES
C     NYEAR      YEARS OF PROJECT LIFE
C
C


C
      REAL MNPV
      DIMENSION AVPQ(20),Z(10),PQAV(10,50),SDPQ(20),RINYR(10,10),
     1RBEYR(10),R(10,10),C(10,10),TEMP(10,50)
      DIMENSION DT(50,3) ,KFL(10),DCT(50),FLIRR(50),JFL(10),NYR(50),
     1WGT(50),WGTS(50,10),FPQ(50),PV(100),RR(100),CFL(50),UBD(3),
     2STATS(5),FREQ(20),PCT(20),CFREQ(50),GENPQ(10,50),KYR(50)
      DIMENSION ITABL(310)
      COMMON SEED
      DATA UBD/1.,17.,1./
      SEED=0.7259
      READ(2,219)(ITABL(I),I=1,310)
  219 FORMAT(15(20I4/),10I4)
C
C     READ THE NUMBER OF STOCHASTIC VARIABLES , YEARS OF PROJECT LIFE
C     AND THE NUMBER OF SIMULATIONS
C
      READ(2,100)NPQ,NYEAR,NO,JJJ
  100 FORMAT(3I5,I2)
      WRITE(3,221)
  221 FORMAT('1'/////' THE NUMBER OF STOCHASTIC VARIABLES, YEARS OF PROJ
     1ECT LIFE'/' AND NUMBER OF SIMULATIONS ARE ----'///)
      WRITE(3,100)NPQ,NYEAR,NO
      NYEAR=NYEAR+1
      TPV=0.0
      TPS=0.0
      DO 40 I=1,NPQ
      AVPQ(I)=0.0
      SDPQ(I)=0.0
      RBEYR(I)=0.0
      DO  40 J=1,NPQ
   40 RINYR(I,J)=0.0
```

```
C
C     READ IN THE MEANS AND STANDARD DEVIATIONS OF
C     THE STOCHASTIC VARIABLES
C
      READ(2,101)(AVPQ(I),I=1,6)
  101 FORMAT(10X,6F10.0)
      IF(NPQ-6)600,600,601
  601 DO 400 K=7,NPQ,6
      K5=K+5
  400 READ(2,101)(AVPQ(I),I=K,K5)
  600 CONTINUE
      WRITE(3,222)
  222 FORMAT(///' THE MEANS AND STANDARD DEVIATIONS OF THE STOCHASTIC VA
     1RIABLES ARE ---'///)
      WRITE(3,609)(AVPQ(I),I=1,NPQ)
  609 FORMAT(3(10X,10F10.2/)//)
      READ(2,101)(SDPQ(I),I=1,6)
      IF(NPQ-6)602,602,603
  603 DO 402 K=7,NPQ,6
      K5=K+5
  402 READ(2,101)(SDPQ(I),I=K,K5)
  602 CONTINUE
      WRITE(3,609)(SDPQ(I),I=1,NPQ)
C
C     READ IN THE CROSS CORRELATION COEFFICIENTS
C
      DO 719 J=1,NPQ
      READ(2,718)NOB,(RINYR(I,J),I=1,NPQ)
  718 FORMAT(2X,I3,15F5.2)
      IF(NOB-15)719,719,720
  720 DO 721 K=16,NOB,15
      K14=K+14
  721 READ(2,722)(RINYR(I,J),I=K,K14)
  722 FORMAT(5X,15F5.2)
  719 CONTINUE
C
C     READ IN THE AUTOCORRELATION COEFFICIENTS
      READ(2,718)NOB,(RBEYR(I),  I=1,NPQ)
      IF(NOB-15)519,519,520
  520 DO 521 K=16,NOB,15
      K14=K+14
  521 READ(2,722)(RBEYR(I),  I=K,K14)
  519 CONTINUE
C
C     FILL OUT UPPER TRIANGLE OF RINYR
C
      MPQ=NPQ-1
      DO 761 I=1,MPQ
      K=I+1
      DO 761 J=K,NPQ
  761 RINYR(J,I)=RINYR(I,J)
C
C     PRINT THE CORRELATION COEFFICIENTS
C
      WRITE(3,223)
  223 FORMAT(///20X,' CROSS CORRELATION COEFFICIENTS'///)
      DO 762 J=1,NPQ
      WRITE(3,731)(RINYR(I,J),I=1,NPQ)
  731 FORMAT(/15X,20F5.2/15X,20F5.2/)
  762 CONTINUE
      WRITE(3,224)
  224 FORMAT(///20X,' AUTOCORRELATION COEFFIENTS'///)
      WRITE(3,732)(RBEYR(I),   I=1,NPQ)
  732 FORMAT(//15X,20F5.2//)
      DO 608 I=1,NPQ
      DO 608 J=1,NPQ
      R(I,J)=0.0
  608 C(I,J)=0.0
C
C     COMPUTE THE VAR-COVAR MATRIX  R
C
      CALL ABVCO(NPQ,SDPQ,RINYR,R)
      WRITE(3,225)
  225 FORMAT('1'/////20X,' THE VARIANCE COVARIANCE MATRIX'/////)
      DO 110 J=1,NPQ
  110 WRITE(3,607)(R(I,J),I=1,NPQ)
  607 FORMAT(/ (5X,10F10.5/))
```

```
C
C     FIND THE SQUARE ROOT MATRIX  C
C
      CALL SQRM(R,NPQ,C)
C
      JYRS=NYEAR
      IFLT=NPQ
      RIN=1.10
      DO 760 I=1,3
      DO 760 J=1,50
  760 DT(J,I)=0.0
C
C     READ IN THE BENEFIT FLOW, THE ASSOCIATED COST FLOW AND THE
C     CAPITAL COST FLOW AND PRINT AS INPUTED
C
      DO 211 I=1,3
      READ(2,207)KFL(I),NOBK,(KYR(J),DCT(J),J=1,6)
  207 FORMAT(3X,2I2,6(I3,F8.0))
      IF(NOBK-6)210,210,208
  208 DO 209 K = 7,NOBK,6
      K5 = K +5
      READ(2,102)(KYR(J),DCT(J),J=K,K5)
  102 FORMAT(7X,6(I3,F8.0))
  209 CONTINUE
  210 KYR(NOBK+1) = KYR(NOBK) + 1
C
C     INTERPOLATE
C
      LL=1
      DO 617 J=1,NOBK
      K= KYR(J+1)-KYR(J)
      DT(LL,I)=DCT(J)
      IF(K-1)617,617,619
  619 ADD=(DCT(J+1)-DCT(J))/K
      L=LL+K-2
      DO 620 M=LL,L
  620 DT(M+1,I)=DT(M,I)+ADD
      LL=L+1
  617 LL=LL+1
      DO 621 J=LL,50
  621 DT(J,I)=DCT(NOBK)
  211 CONTINUE
C
C     PRINT IN CONSECUTIVE ORDER THE FULL FLOWS OF BENEFITS,
C     ASSOCIATED COSTS AND CAPITAL COSTS
C
  775 WRITE(3,604)
  604 FORMAT(///////' INPUT FLOWS TO BE DISCOUNTED'////)
      WRITE(3,605)(DT(I,1),I=1,JYRS)
  605 FORMAT(////' BENEFITS'6X, 6(10F10.0/15X)//)
      WRITE(3,623)(DT(I,2),I=1,JYRS)
  623  FORMAT(//' ASSOC COSTS'3X, 6(10F10.0/15X)//)
      WRITE(3,624)(DT(I,3),I=1,JYRS)
  624 FORMAT(//' CAPITAL COSTS'1X, 6(10F10.0/15X))
C
C     CALCULATE THE NET CASH FLOW
C
  763 DO 212 I =1,JYRS
      FLIBR(I) =DT(I,1)-DT(I,2)-DT(I,3)
  212 CONTINUE
C
C
C     READ IN WEIGHTS CORRESPONDING TO STOCHASTIC VARIABLES AND PRINT
C
      DO 1 I =1, IFLT
      READ(2,207)JFL(I),NOBS,(NYR(J),WGT(J),J=1,6)
      IF(NOBS-6)2,2,3
    3 DO 4 K=7,NOBS,6
      K5 =K+5
      READ(2,102)(NYR(J),WGT(J),J=K,K5)
    4 CONTINUE
    2 NYR(NOBS+1)=NYR(NOBS)+1
C
C     INTERPOLATE
C
      LL=1
      DO 5 J=1,NOBS
```

```
      K= NYR(J+1)-NYR(J)
      WGTS(LL,I)= WGT(J)
      IF(K-1)5,5,6
    6 ADD=(WGT(J+1)-WGT(J))/K
      L=LL +K-2
      DO 7 M=LL,L
    7 WGTS(M+1,I)= WGTS(M,I)+ADD
      LL =L +1
    5 LL=LL+1
C
      DO 8 J=LL,50
    8 WGTS(J,I)=WGT(NOBS)
    1 CONTINUE
C
C     PRINT FULL MARIX OF WEIGHTS
C
  764 WRITE(3,103)
  103 FORMAT('1',////30X,' MATRIX OF WEIGHTS CORRRSPUNDING TO'/30X,
     1'THE STOCHASTIC VARIABLES'///6X,' FLOW NO'//)
      DO 9 J=1,IFLT
      WRITE(3,104)JFL(J),(WGTS(I,J),I=1,JYRS)
  104 FORMAT(/8X,I2,5X,6(10F10.0/15X))
    9 CONTINUE
C
C     CARRY OUT SIMULATION FOR NO RUNS
C
      DO 901 JJ=1,JJJ
      TPV=0.
      TPS=0.
      DO 204 NS=1,NO
      DCFL=0.0
      DO 200 J=1,NYEAR
  200 FPQ(J)=0.0
      DO 23 I=1,NPQ
   23 CALL GAUSS(IX,1.,0.,Z(I))


      DO 30 I=1,NPQ
      GENPQ(I,1)=AVPQ(I)


      DO 30 J=1,NPQ
   30 GENPQ(I,1)=GENPQ(I,1)+Z(J)*C(I,J)

      DO 31 M=2,NYEAR
      MM=M-1
      DO 31 I=1,NPQ
      PQAV(I,M)=RBEYR(I)*(GENPQ(I,MM)-AVPQ(I))+AVPQ(I)
      DO 32 L=1,NPQ
   32 CALL GAUSS(IX,1.,0.,Z(L))
      GENPQ(I,M)=PQAV(I,M)
      DO 31 J=1,NPQ
   31 GENPQ(I,M)=GENPQ(I,M)+Z(J)*C(I,J)
C
C     GENERATE THE NET CASH FLOW FOR RUN NS
C
      DO 201 J=1,NYEAR
      DO 202 K=1,NPQ
      TEMP(K,J)=WGTS(J,K)*GENPQ(K,J)
  202 FPQ(J)=FPQ(J)+TEMP(K,J)
  201 CFL(J)=FPQ(J)+FLIRR(J)



C
C     COMPUTE  IRR
C
      CALL ALIRR(CFL,NYEAR,0.0001,10,RRT)
      RR(NS)=RRT*100.0

C
C     DISCOUNT THE NET CASH FLOW
C
      DCFL=CFL(1)
      DO  203 I=2,NYEAR
      IT=I-1
```

```
203 DCFL=DCFL+CFL(I)/RIN**IT

    TPV=TPV + DCFL
    TPS=TPS + DCFL*DCFL
204 PV(NS)=DCFL
C
C   TABULATE, GRAPH AND PRINT RESULTS
C
    INT1=UBO(2)-1
    CALL TAB1(RR,RR,1,UBO,FREQ,PCT,STATS,NO,1,SINT)
    CFREQ(1)=PCT(1)/100.
    DO 900 I=2,INT1
    PCT(I)=PCT(I)/100.
900 CFREQ(I)=CFREQ(I-1)+PCT(I)
    AMEAN=STATS(4)-SINT/2.
    CALL ABGPH(CFREQ,CFREQ,INT1,AMEAN,SINT)
    MNPV=TPV/NO
    SDPV=SQRT(ABS((TPS-TPV*TPV/NO)/(NO-1)))
    WRITE(3,215)MNPV,SDPV
215 FORMAT(//' EXPECTED PV'F20.2//' STANDARD DEVIATION'F13.2//)
    CALL ABPGZ(ITABL,MNPV,SDPV,A)
    WRITE(3,220)A
220 FORMAT(' PROB OF PV .GT. ZERO ='F9.0,' PER CENT')
901 CONTINUE
    CALL EXIT
    END
```

THE NUMBER OF STOCHASTIC VARIABLES, YEARS OF PROJECT LIFE
AND NUMBER OF SIMULATIONS ARE ----

10 36 100

THE MEANS AND STANDARD DEVIATIONS OF THE STOCHASTIC VARIABLES ARE ---

-6.65	-16.67	-0.91	0.62	0.60	0.30	0.83	0.62	0.74	0.68
1.25	2.72	0.16	0.09	0.08	0.03	0.18	0.08	0.10	0.09

CROSS CORRELATION COEFFICIENTS

1.00	0.24	0.24	0.34	0.23	0.27	0.12	0.12	0.12	0.12
0.24	1.00	1.00	0.20	0.87	0.85	0.96	0.96	0.96	0.96
0.24	1.00	1.00	0.20	0.87	0.85	0.96	0.96	0.96	0.96
0.34	0.20	0.20	1.00	0.09	-0.01	0.16	0.16	0.16	0.16
0.23	0.87	0.87	0.09	1.00	0.90	0.92	0.92	0.92	0.92
0.27	0.85	0.85	-0.01	0.90	1.00	0.89	0.89	0.89	0.89
0.12	0.96	0.96	0.16	0.92	0.89	1.00	1.00	1.00	1.00
0.12	0.96	0.96	0.16	0.92	0.89	1.00	1.00	1.00	1.00
0.12	0.96	0.96	0.16	0.92	0.89	1.00	1.00	1.00	1.00
0.12	0.96	0.96	0.16	0.92	0.89	1.00	1.00	1.00	1.00

AUTOCORRELATION COEFFIENTS

-0.26	0.35	0.35	0.19	0.13	-0.18	0.46	0.46	0.46	0.46

THE VARIANCE COVARIANCE MATRIX

1.56250	0.81600	0.04800	0.03825	0.02300	0.01013	0.02700	0.01200	0.01500	0.01350
0.81600	7.39840	0.43520	0.04896	0.18931	0.06936	0.47002	0.20890	0.26112	0.23501
0.04800	0.43520	0.02560	0.00288	0.01114	0.00408	0.02765	0.01229	0.01536	0.01382
0.03825	0.04896	0.00288	0.00810	0.00065	-0.00003	0.00259	0.00115	0.00144	0.00130
0.02300	0.18931	0.01114	0.00065	0.00640	0.00216	0.01325	0.00589	0.00736	0.00662
0.01013	0.06936	0.00408	-0.00003	0.00216	0.00090	0.00481	0.00214	0.00267	0.00240
0.02700	0.47002	0.02765	0.00259	0.01325	0.00481	0.03240	0.01440	0.01800	0.01620
0.01200	0.20890	0.01229	0.00115	0.00589	0.00214	0.01440	0.00640	0.00800	0.00720
0.01500	0.26112	0.01536	0.00144	0.00736	0.00267	0.01800	0.00800	0.01000	0.00900
0.01350	0.23501	0.01382	0.00130	0.00662	0.00240	0.01620	0.00720	0.00900	0.00810

THE SQUARE ROOT MATRIX C

1.25000	0.65280	0.03840	0.03060	0.01840	0.00810	0.02160	0.00960	0.01200	0.01080
0.00000	2.64050	0.15532	0.01098	0.06715	0.02427	0.17266	0.07674	0.09592	0.08633
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.08392	-0.00777	-0.00645	0.00043	0.00019	0.00024	0.00021
0.00000	0.00000	0.00000	0.00000	0.03863	0.00858	0.03262	0.01450	0.01812	0.01631
0.00000	0.00000	0.00000	0.00000	0.00000	0.01142	0.01438	0.00639	0.00799	0.00719
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.02916	0.01296	0.01620	0.01458
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

C TRANSPOSE C
THIS SHOULD EQUAL THE VARIANCE COVARIANCE MATRIX

1.56250	0.81600	0.04800	0.03825	0.02300	0.01013	0.02700	0.01200	0.01500	0.01350
0.81600	7.39840	0.43520	0.04896	0.18931	0.06936	0.47002	0.20890	0.26112	0.23501
0.04800	0.43520	0.02560	0.00288	0.01114	0.00408	0.02765	0.01229	0.01536	0.01382
0.03825	0.04896	0.00288	0.00610	0.00065	-0.00003	0.00259	0.00115	0.00144	0.00130
0.02300	0.18931	0.01114	0.00065	0.00640	0.00216	0.01325	0.00589	0.00736	0.00662
0.01013	0.06936	0.00408	-0.00003	0.00216	0.00090	0.00481	0.00214	0.00267	0.00240
0.02700	0.47002	0.02765	0.00259	0.01325	0.00481	0.03240	0.01440	0.01800	0.01620
0.01200	0.20890	0.01229	0.00115	0.00589	0.00214	0.01440	0.00640	0.00800	0.00720
0.01500	0.26112	0.01536	0.00144	0.00736	0.00267	0.01800	0.00800	0.01000	0.00900
0.01350	0.23501	0.01382	0.00130	0.00662	0.00240	0.01620	0.00720	0.00900	0.00810

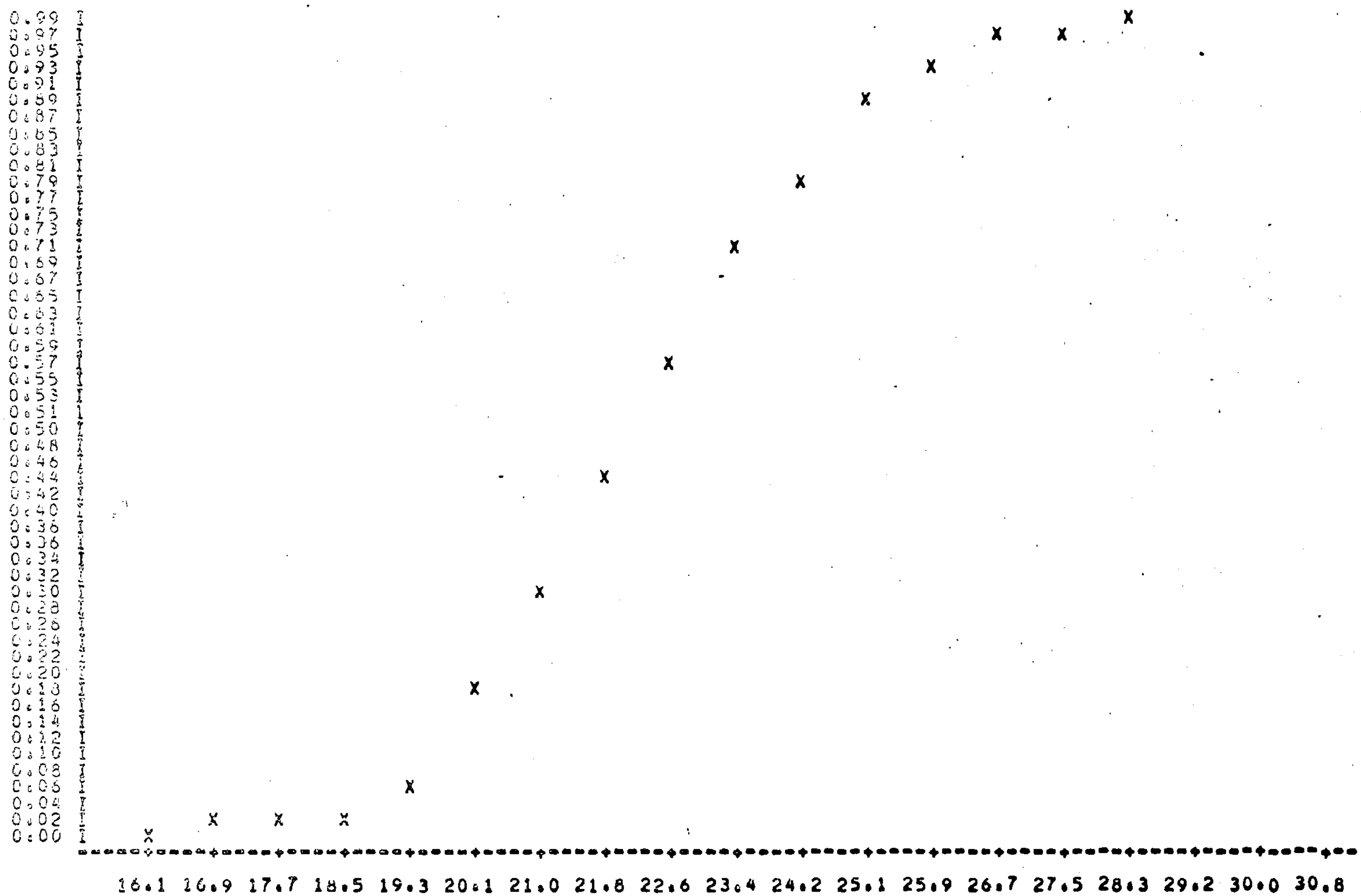
INPUT FLOWS TO BE DISCOUNTED

BENEFITS	0.	16387.	26359.	25282.	12405.	12490.	11244.	11244.	11244.	11244.
	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.
	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.	11244.
	11244.	11244.	11244.	11244.	11244.	8436.	0.			
ASSOC COSTS	0.	23203.	104494.	161629.	184955.	201780.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.	217304.
	217304.	217304.	217304.	217304.	217304.	187674.	0.			
CAPITAL COSTS	377744.	34946.	34889.	5695.	3427.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.			

MATRIX OF WEIGHTS CORRRSPONDING TO
THE STOCHASTIC VARIABLES

FLOW NO										
2	0.	9368.	7573.	1082.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	-18023.	0.			
4	0.	4580.	3233.	2020.	1866.	1751.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	-13470.	0.			
6	0.	88020.	122762.	142262.	166424.	188770.	178299.	178299.	178299.	178299.
	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.
	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.	178299.
	178299.	178299.	178299.	178299.	178299.	-80551.	0.			
11	0.	0.	44779.	80978.	86150.	86150.	86150.	86150.	86150.	86150.
	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.
	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.	86150.
	86150.	86150.	86150.	86150.	86150.	86150.	0.			
13	0.	0.	92087.	166530.	177166.	177166.	177166.	177166.	177166.	177166.
	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.
	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.	177166.
	177166.	177166.	177166.	177166.	177166.	177166.	0.			
15	0.	0.	34569.	62512.	66505.	66505.	66505.	66505.	66505.	66505.
	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.
	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.	66505.
	66505.	66505.	66505.	66505.	66505.	66505.	0.			
18	0.	0.	13740.	23439.	29499.	35157.	40410.	40410.	40410.	40410.
	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.
	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.	40410.
	40410.	40410.	40410.	40410.	40410.	40410.	0.			
20	0.	0.	18183.	31018.	39037.	46524.	53476.	53476.	53476.	53476.
	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.
	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.	53476.
	53476.	53476.	53476.	53476.	53476.	53476.	0.			
22	0.	0.	1374.	2344.	2950.	3516.	4041.	4041.	4041.	4041.
	4041.	4041.	4041.	4041.	4041.	4041.	4041.	4041.	4041.	4041.
	4041.	4041.	4041.	4041.	4041.	4041.	404.	4041.	4041.	4041.
	4041.	4041.	4041.	4041.	4041.	4041.	0.			
24	0.	0.	195346.	333234.	419420.	499851.	574526.	574526.	574526.	574526.
	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.
	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.	574526.
	574526.	574526.	574526.	574526.	574526.	574526.	0.			

CUMULATIVE PROBABILITY DISTRIBUTION OF THE
INTERNAL RATE OF RETURN



INTERNAL RATE OF RETURN

0.00 0.02 0.02 0.02 0.06 0.18 0.30 0.43 0.57 0.72 0.80 0.89 0.93 0.97 0.97 0.99

EXPECTED PV 1267028.89

STANDARD DEVIATION 177681.24

PROB OF PV .GT. ZERO = 100. PER CENT

APPENDIX VIII

TUAP - LISTING OF PROGRAM, INPUT DATA AND OUTPUT

```
C
C      A MONTE CARLO SIMULATION OF THE TUAPEKA RURAL WATER SUPPLY SCHEME
C
C      THIS PROGRAM IS A SIMULATION MODEL OF A STOCK WATER SUPPLY
C      SCHEME .  IT CALCULATES THE EXPECTED VALUES AND STANDARD DEVIATIO
C      OF THE NET PRESENT VALUE, BENEFIT COST RATIO, AND INTERNAL RATE
C      OF RETURN FOR THE SCHEME.
C      THE PROGRAM ALSO PRINTS OUT A GRAPH OF THE CUMMULATIVE PROBABILIT
C      DISTRABUTION OF THE IRR.
C
C      THE DIMENSIONED VARIABLES ARE ---
C
C      AOO    =  ASSOCIATED COSTS ON-FARM
C      FL     =  NET BENEFIT FLOW
C      FLC    =  CAPITAL COSTS PLUSS ASSOCIATED COSTS
C      OFSC   =  ON-FARM SAVED COSTS
C      PVC    =  DISCOUNTED COST FLOW
C      PVR    =  DISCOUNTED BENEFIT FLOW
C      ROR    =  ARRAY OF SIMULATED INTERNAL RATES OF RETURN
C      SCAC   =  SCHEME ASSOCIATED COSTS
C      SFF    =  CAPITAL COSTS OFF-FARM
C      SOF    =  COST OF STRUCTURES ON-FARM
C      SUB    =  EXTRA BREEDING COWS (SU)
C      SUBCB  =  ACCUMULATIVE BREEDING CATTLE (SU)
C      SUF    =  EXTRA FATTENING CATTLE (SU)
C      SUFCB  =  ACCUMULATIVE FATTENING CATTLE (SU)
C      SUS    =  EXTRA SHEEP (SU)
C      SUSCB  =  ACCUMULATIVE SHEEP NUMBERS (SU)
C      TCAC   =  ASSOCIATED COSTS
C      SVS    =  SALVAGE OF STRUCTURES
C      SVSKB  =  BREEDING CATTLE NUMBERS (SU) SALVAGED
C      SVSKF  =  FATTENING CATTLE NUMBERS (SU) SALVAGED
C      SVSKS  =  SHEEP NUMBERS (SU) SALVAGED AT END OF SCHEME
C      TCC    =  CAPITAL COSTS
C      TGR    =  GROSS REVENUE
C
C      OTHER INPUT VARIABLES
C
C      AL     =  LBS OF LAMB MEAT PER (SU)
C      ALP    =  LAMB PROCESSING COST PER (SU)
C      AMP    =  MUTTON PROCESSING COST PER (SU)
C      BL     =  LBS OF BEEF PER BULL
C      CC     =  LBS OF BEEF PER CULL COW
C      BLS    =  BULL REPLACEMENT PER (SU) BEEF BREEDING
C      BOF    =  OTHER ASSOCIATED COSTS BEEF BREEDING
C      BRP    =  BEEF BREEDING PROCESSING COST PER (SU)
C      FOF    =  ON-FARM BEEF FATTENING ASSOCIATED COSTS
C      HF     =  LBS OF BEEF PER HIEFER
C      NO     =  NUMBER OF SIMULATIONS
C      NYRS   =  YEARS OF PROJECT LIFE
C      SPOF   =  ON-FARM SHEEP ASSOCIATED COSTS
C      WL     =  KILOS OF WOOL PER (SU)
C      WLP    =  KILOS OF WOOL PER (SU)
C      WR     =  LBS OF BEEF PER WEANER
C      WT     =  WEIGHTING FACTOR TO BRING SAMPLE UP TO TOTAL POPULATION
C
       REAL MNPV
       DIMENSION FL(100),TGR(100),TCC(100),TCAC(100),SUS(100),SUB(100),
      1SUF(100),SUSCB(100),SUFCB(100),SVS(100),OFSC(100),SUBCB(100)
       DIMENSION SCAC(100),AOO(100),SOF(100),SVSKS(100),SVSKB(100),
      1SVSKF(100),SFF(100),FLC(100),PVC(100),PVR(100)
       DIMENSION FREQ(20),PCT(20),STATS(5),UBO(3),CFREQ(20),ROR(100)
       DATA UBO/1.,17.,1./,TPV,TPV2/2*0./
       DATA BCT/0./,BCT2/0./,PVCT,PVRT/2*0./
        IX=3521
```

```
C
C     READ IN SCHEME LIFE, NO. OF SIMULATIONS AND OTHER PARAMETERS
C
      READ(2,503)NYRS,NO,WT,WL,AL,AM,WR,HR,CC,BL,WLP,ALP,AMP,BRP,BFP,
     1BLS,BOF,SPOF,FOF
  503 FORMAT(2I3,F4.2,7F10.6/8F10.6/F10.6)
      WRITE(3,103)
  103 FORMAT('1'////20X,' SCHEME PARAMETERS'///3X,' NYRS  NO  WT  WL
     1L  AM   WR   HR   CC   BL'/3X,' WLP   ALP AMP BRP BFP BLS BOF'/ 3X,'
     2OF'///)
      WRITE(3,503)NYRS,NO,WT,WL,AL,AM,WR,HR,CC,BL,WLP,ALP,AMP,BRP,BFP,
     1BLS,BOF,SPOF,FOF
C
C     READ IN DATA FOR CASH FLOWS.
C
      READ(2,50)(SUS(I),SUB(I),SUF(I),SUSCB(I),SUBCB(I),SUFCB(I),SVS(I)
     1OFSC(I),SCAC(I),AOO(I),SOF(I),SVSKS(I),SVSKB(I),SVSKF(I),SFF(I)
     2,I=1,36)
   50 FORMAT(14F5.0,F6.0)
      WRITE(3,104)
  104 FORMAT(////25X,' SCHEME FLOWS'///3X,' SUS   SUB   SUF   SUSCB   SUBC
     1   SUFCB   SVS   OFSC   SCAC   AOO'/3X,' SOF   SVSKS   SVSKB   SVSKF   SFF
     2///)
      WRITE(3,79)(SUS(I),SUB(I),SUF(I),SUSCB(I),SUBCB(I),SUFCB(I),SVS(I
     1,OFSC(I),SCAC(I),AOO(I),SOF(I),SVSKS(I),SVSKB(I),SVSKF(I),SFF(I)
     2,I=1,36)
   79 FORMAT(10F10.0/5F10.0)
      DO 999 III=1,10
      TPV =0.0
      TPV2=0.0
      PVRT=0.0
      PVCT=0.0
      BCT =0.0
      BCT2=0.0
      DO 102M=1,NO
      DO 80 I=1,NYRS
C
C     CALCULATE NET REVENUE OF SCHEME.
C
C     GROSS REVENUE FOR SHEEP ENTERPRISE YEAR(I)
C
C
      CALL TRICR(IX,.44,.55,.89,OBS,RN)
      RNW=RN
      WOOL=WL*OBS
      ISOT =IX
      CALL TRICR(IX,.19,.25,.38,OBS,RN)
      RNS=RN
      ALAB=AL*OBS
      IX =ISOT
      CALL TRIAN(IX,.105,.13,.185,OBS)
      AMUT=AM*OBS
      GRS=WOOL+ALAB+AMUT
      SPB=GRS*SUSCB(I)
C
C     GROSS REVENUE FOR BEEF BREEDING ENTERPRISE YEAR(I)
C
      IBOT =IX
      CALL TRIAN(IX,47.,82.,122.,OBS)
      WNR=WR*OBS
      IX =IBOT
      CALL TRIAN(IX,32.,67.,107.,OBS)
      HFR=HR*OBS
      IX =IBOT
      CALL TRIAN(IX,78.,114.,150.,OBS)
      CULC=CC*OBS
      IX =IBOT
      CALL TRIAN(IX,169.,252.,334.,OBS)
      BUL=BL*OBS
      GRB=WNR+HFR+CULC+BUL
      BRB=GRB*SUBCB(I)
      CONTINUE
C
C     GROSS REVENUE FOR BEEF FATTENING ENTERPRISE YEAR(I)
C
      IX =IBOT
      CALL TRIAN(IX,94.5,139.,184.5,OBS1)
```

```
      GRF=OBS1*.2475
      BFB=GRF*SUFCB(I)
C     SALVAGE OF STOCK
      RNC=(RNS+RNW)/2.
      CALL TRICR(-1,4.,6.,10.,OBS,RNC)
      SSV=SVSKS(I)*OBS
      IX=IBOT
      CALL TRIAN(IX,10.,16.6,23.3,OBS)
      BSV=SVSKB(I)*OBS
      IX=IBOT
      CALL TRIAN(IX,8.7,17.5,27.5,OBS)
      FSV=SVSKF(I)*OBS
      TGR(I)=(SPB+BRB+BFB+SVS(I)+OFSC(I)+SSV+BSV+FSV)*WT
C
C     CAPITAL COSTS  YEAR(I)
C
      CALL TRICR(-1,4.,6.,10.,OBS,RNC)
      CS=SUS(I)*OBS
      IX =IBOT
      CALL TRIAN(IX,10.,16.6,23.3,OBS)
      CB=SUB(I)*OBS
      IX =IBOT
      CALL TRIAN(IX,47.,82.,122.,OBS2)
      CF=SUF(I)*OBS2*.2475
      OFC=(CS+CB+CF+SOF(I))*WT
      TCC(I)=OFC+SFF(I)
C
C     ASSOCIATED COSTS OFF FARM YEAR(I)
C
      CALL TRIAN(IX,.105,.11,.12,OBS)
      WP=WLP*OBS
      ACS=(WP+ALP+AMP)*SUSCB(I)
      ACB=BRP*SUBCB(I)
      ACF=BFP*SUFCB(I)
      TACF=(ACS+ACB+ACF)*WT
      TFFA=TACF+SCAC(I)
C
C     ASSOCIATED COSTS ON FARM YEAR(I)
C
      IX =IBOT
      CALL TRIAN(IX,200.,300.,400.,OBS)
      BAC=BLS*OBS
      AOB=(BAC+BOF)*SUBCB(I)
      AOS=SPOF*SUSCB(I)
      AOF=FOF*SUFCB(I)
      TOFA=(AOB+AOS+AOF+AOO(I))*WT
C
C     TOTAL ASSOCIATED COSTS YEAR (I)
C
      TCAC(I)=TFFA+TOFA
C
C     TOTAL COSTS YEAR (I)
C
      FLC(I) = TCC(I) + TCAC(I)
   80 FL(I)=TGR(I)-FLC(I)
C
C     CALCULATION OF NPV
C
      PV=FL(1)
      DO 905 I=2,NYRS
  905 PV=PV+FL(I)/((1.1)**(I-1))
      TPV=TPV+PV
      TPV2=TPV2+PV*PV
C
C     CALCULATION OF BENEFIT COST RATIO
C
      DO 400 I = 2, NYRS
      PVR(I) = TGR(I)/((1.1)**(I-1))
      PVRT=PVRT+PVR(I)
      PVC(I) = FLC(I)/((1.1)**(I-1))
      PVCT=PVCT+PVC(I)
  400 CONTINUE
      BCRT = (PVRT + TGR(1))/(PVCT + FLC(1))
      BCT = BCT + BCRT
      BCT2= BCT2 +BCRT*BCRT
C
C     CALCULATION OF  IRR
```

```
C
      CALL ALIRR(FL,NYRS,0.0001,10,RRT)
  102 ROR(M)=RRT*100
C
C     TABULATE AND PRINT RESULTS
C
      INT1=UBO(2)-1
      CALL TABA(ROR,ROR,1,UBO,FREQ,PCT,STATS,NO,1,SINT)
      CFREQ(1)=PCT(1)/100.
      DO 900 I=2,INT1
      PCT(I)=PCT(I)/100.
  900 CFREQ(I)=CFREQ(I-1)+PCT(I)
      AMEAN=STATS(4)-SINT/2.
      CALL ABGPH(PCT,CFREQ,INT1,AMEAN,SINT)
      WRITE(3,64)
   64 FORMAT(' GRAPH OF INTERNAL RATE OF RETURN WHERE THE CUMULATIVE '/
     1' PROBABILITY DISTRABUTION IS SHOWN BY ... '/
     2' AND THE DENSITY FUNCTION BY ***')
      MNPV=TPV/NO
      SDPV=SQRT(ABS((TPV2-TPV*TPV/NO)/(NO-1)))
      WRITE(3,65)
   65 FORMAT('1'5X ' SUMMARY  OF  RESULTS'/6X '----------------------'
     1///)
      WRITE(3,500)(STATS(I),I=2,5)
  500 FORMAT(' EXPECTED IRR'F19.2//' STANDARD DEVIATION'F13.2//
     1' RANGE'F26.1' TO'F5.1//)
      WRITE(3,505)MNPV,SDPV
  505 FORMAT(' EXPECTED PV'F20.2//' STANDARD DEVIATION'F13.2//)
      BCE = BCT/NO
      SDBCT = SQRT(ABS((BCT2 - BCT*BCT/NO)/(NO-1)))
      WRITE(3,450)BCE,SDBCT
  450 FORMAT(' EXPECTED B/C RATIO'F13.2// ' STANDARD DEVIATION'F13.2///
     1//)
      WRITE (3,501)
  501 FORMAT(   9X'FREQUENCY DISTRIBUTION OF'/11X'INTERNAL RATE OF RETU
     1N'//7X'CLASS'5X'FREQUENCY'5X'CUMULATIVE'/8X'MEAN'18X'PROBABILITY'
      DO 901 I=2,INT1
      AMEAN=AMEAN+SINT
      NFRQ=FREQ(I)
  901 WRITE (3,502)AMEAN,NFRQ,CFREQ(I)
  502 FORMAT(F11.1,10X,I2,F15.2)
  999 CONTINUE
      CALL EXIT
      END
```

SCHEME PARAMETERS

NYRS NO WT WL AL AM WR HR CC BL
WLP ALP AMP BRP BFP BLS BOF
FOF

361002.40	4.782610	21.678260	8.130441	0.062700	0.036530	0.021920	0.000880
4.782610	1.300700	0.406530	1.792550	3.682500	0.000880	1.925500	1.332180
1.982500							

SCHEME FLOWS

SUS SUB SUF SUSCB SUBCB SUFCB SVS UFSC SCAC AOO
SOF SVSKS SVSKB SVSKF SFF

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
41528.	0.	0.	0.	278077.					
3903.	1908.	1612.	0.	0.	0.	390.	6438.	5203.	7500.
14561.	0.	0.	0.	0.					
3155.	1347.	2750.	3903.	1908.	1612.	0.	10983.	7903.	9576.
14537.	0.	0.	0.	0.					
451.	842.	3461.	7058.	3255.	2750.	0.	10534.	7903.	10304.
2373.	0.	0.	0.	0.					
0.	786.	4125.	7509.	4097.	3461.	0.	5169.	7903.	11031.
1428.	0.	0.	0.	0.					
0.	730.	4741.	7509.	4883.	4125.	0.	5204.	7903.	11150.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
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0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.

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0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
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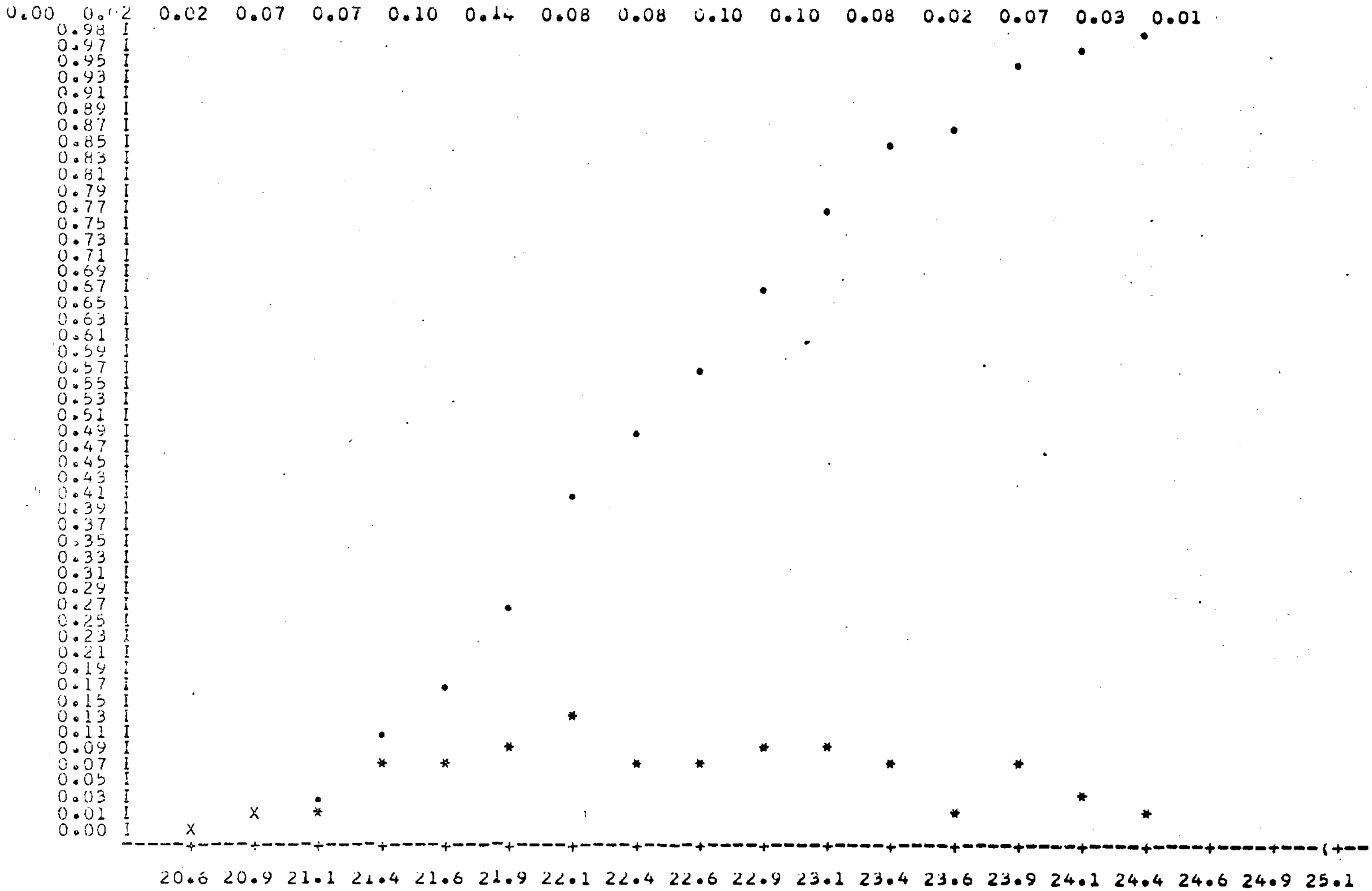
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	4741.	7509.	5613.	4741.	0.	4685.	7903.	11221.
0.	0.	0.	0.	0.					
0.	0.	0.	7509.	5613.	4741.	0.	3515.	5203.	0.
0.	7509.	5613.	0.	0.					

CUMULATIVE PROBABILITY DISTRIBUTION OF THE
INTERNAL RATE OF RETURN



INTERNAL RATE OF RETURN

GRAPH OF INTERNAL RATE OF RETURN WHERE THE CUMULATIVE
PROBABILITY DISTRABUTION IS SHOWN BY ...
AND THE DENSITY FUNCTION BY ***

SUMMARY OF RESULTS

EXPECTED IRR	22.61
STANDARD DEVIATION	0.84
RANGE	20.8 TO 24.5
EXPECTED PV	1254013.50
STANDARD DEVIATION	67585.07
EXPECTED B/C RATIO	1.39
STANDARD DEVIATION	0.01

FREQUENCY DISTRIBUTION OF
INTERNAL RATE OF RETURN

CLASS MEAN	FREQUENCY	CUMULATIVE PROBABILITY
20.9	2	0.02
21.1	2	0.04
21.4	7	0.10
21.6	7	0.18
21.9	10	0.28
22.1	14	0.42
22.4	8	0.50
22.6	8	0.58
22.9	10	0.67
23.1	10	0.77
23.4	8	0.85
23.6	2	0.87
23.9	7	0.94
24.1	3	0.97
24.4	1	0.98

APPENDIX IX

SUBROUTINES USED BY THE VARIOUS PROGRAMS

```
      SUBROUTINE ALIRR(FLOW,N,ERR,NST,RRT)
C
C     FLOW = THE CASH FLOW TO BE OPERATED ON
C     N        LENGTH OF THE FLOW
C     ERR = ERROR ACCEPTABLE FOR NPV=0 EG 0.0001
C     NST      NUMBER OF TIMES ALLOWED FOR CONVERGENCE EG 10
C     RRT      VALUE RETURNED  IRR
C
      DIMENSION FLOW(1)
      ADD1 =10000.
      N1 = N-1
      X= .1
      DO 4 J=1,NST
      PVL =FLOW(1)
      DO 1 I= 2,N1
    1 PVL=PVL+FLOW(I)*((1.+X)**(1-I))
      PVL =PVL +FLOW(N)/(X*(1+X)**(N-2))
      GRAD = 0.
      DO 2 I= 2,N1
    2 GRAD = GRAD +(FLOW(I)*(1-I))/((1.+X)**I)
      GRAD = GRAD - FLOW(N)*(1.+X*(N-1))/((X**2)*(1.+X)**(N-1))
      ADD = PVL/ GRAD
      IF(ABS(GRAD)-0.01)5,5,6
    6 ADD1 =ADD
      GRAD1 =GRAD
      X=X -ADD
      IF(ABS(ADD)-ERR)3,3,4
    4 CONTINUE
      WRITE(1,501)
  501 FORMAT(' IRR HAS NOT CONVERGED ')
    3 RRT =X
      RETURN
    5 WRITE(1,502)
  502 FORMAT(' IRR DOES NOT EXIST, IT HAS BEEN SET TO ZERO ')
      RRT =0.
      RETURN
      END
```

```
      SUBROUTINE ABPGZ(ITABL, PV , SD , A )
C
C     CALCULATES THE PROBABILITY THAT THE PRESENT VALUE IS
C     GREATER THAN ZERO
C     ITABL  =   NORMAL TABLES
C     PV         EXPECTED PRESENT VALUE
C     SD         STANDARD DEVIATION
C     A          PROBABILITY OF VALUE GREATER THAN ZERO
C
      DIMENSION ITABL(310)
      Z=PV/SD
      A=Z*100.
      IT=A
      IF(Z)10,20,30
   20 A=50.
      RETURN
   30 IF(IT-310)40,40,50
   50 A=100.
      RETURN
   40 I=ITABL(IT)
      A=I/100. +50.
      RETURN
   10 IT=-IT
      IF(IT-310)60,60,70
   70 A=0.0
      RETURN
   60 I=ITABL(IT)
      A=50.-I/100.
      RETURN
      END
```

```
      SUBROUTINE GMTRA(A,R,N,M)
      DIMENSION A(1),R(1)
      IR=0
      DO 10 I=1,N
      IJ=I-N
      DO 10 J=1,M
      IJ=IJ+N
      IR=IR+1
   10 R(IR)=A(IJ)
      RETURN
      END
```

```
      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
      R(IR)=0
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
```

```
      SUBROUTINE ABRGH(PV,SDV,ITABL,PROB,N)
C     CALCULATES AND GRAPHS THE PROBABILITY DISTRIBUTION OF THE
C     INTERNAL RATE OF RETURN
C     PV     = ARRAY OF PRESENT VALUES - INTEREST RATES 1TO N
C     SDV      ARRAY OF STANDARD DEVIATIONS SAME RATES
C     ITABL    ARRAY CONTAINING THE Z TABLES * 10000
C     PROB     CUMULATIVE PROBABILITY DISTRIBUTION OF IRR
C     N        NUMBER OF OBSERVATIONS
      DIMENSION PV(1),SDV(1),PROB(1),ITABL(1)
      DO 610 I=1,N
      IZ=0
      IZ=((0-PV(I))/SDV(I))*100
      IF(IZ)611,617,612
  617 PROB(I)=0.50
      GO TO 610
  612 IF(IZ-310)616,616,614
  614 PROB(I)=1.0
      GO TO 610
  616 PROB(I)=ITABL(IZ)*0.0001+0.5
      GO TO 610
  611 IZ= -1*IZ
      IF(IZ-310)615,615,613
  613 PROB(I)=0.00
      GO TO 610
  615 PROB(I)=(ITABL(IZ)*0.0001-0.5)
      PROB(I)=-PROB(I)
  610 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TAB1(A,S,NOVAR,UBO,FREQ,PCT,STATS,NO,NV,SINT)
      DIMENSION A(1),S(1),UBO(3),FREQ(1),PCT(1),STATS(5)
      DIMENSION WBO(3)
      DO 5 I=1,3
    5 WBO(I)=UBO(I)
C         CALCULATE MIN AND MAX
      VMIN=1.0E38
      VMAX=-1.0E38
      IJ=NO*(NOVAR-1)
      DO 30 J=1,NO
      IJ=IJ+1
      IF(S(J)) 10,30,10
   10 IF(A(IJ)-VMIN) 15,20,20
   15 VMIN=A(IJ)
   20 IF(A(IJ)-VMAX) 30,30,25
   25 VMAX=A(IJ)
   30 CONTINUE
      STATS(4)=VMIN
      STATS(5)=VMAX
C         DETERMINE LIMITS
      IF(UBO(1)-UBO(3)) 40,35,40
   35 UBO(1)=VMIN
      UBO(3)=VMAX
   40 INN=UBO(2)
C         CLEAR OUTPUT AREAS
      DO 45 I=1,INN
      FREQ(I)=0.0
   45 PCT(I)=0.0
      DO 50 I=1,3
   50 STATS(I)=0.0
C         CALCULATE INTERVAL SIZE
      SINT=ABS((UBO(3)-UBO(1))/(UBO(2)-2.0))
C         TEST SUBSET VECTOR
      SCNT=0.0
      IJ=NO*(NOVAR-1)
      DO 75 J=1,NO
      IJ=IJ+1
      IF(S(J)) 55,75,55
   55 SCNT=SCNT+1.0
C         DEVELOP TOTAL AND FREQUENCIES
      STATS(1)=STATS(1)+A(IJ)
      STATS(3)=STATS(3)+A(IJ)*A(IJ)
      TEMP=UBO(1)-SINT
      INTX=INN-1
      DO 60 I=1,INTX
      TEMP=TEMP+SINT
      IF(A(IJ)-TEMP) 70,60,60
   60 CONTINUE
      IF(A(IJ)-TEMP) 75,65,65
   65 FREQ(INN)=FREQ(INN)+1.0
      GO TO 75
   70 FREQ(I)=FREQ(I)+1.0
   75 CONTINUE
C         CALCULATE RELATIVE FREQUENCIES
      DO 80 I=1,INN
   80 PCT(I)=FREQ(I)*100.0/SCNT
C         CALCULATE MEAN AND STANDARD DEVIATION
      IF(SCNT-1.0) 85,85,90
   85 STATS(2)=0.0
      STATS(3)=0.0
      GO TO 95
   90 STATS(2)=STATS(1)/SCNT
      STATS(3)=SQRT(ABS((STATS(3)-STATS(1)*STATS(1)/SCNT)/(SCNT-1.0)))
   95 DO 100 I=1,3
  100 UBO(I)=WBO(I)
      RETURN
      END
```

```
      SUBROUTINE SQRM(V,M,C)
C
C     SUBROUTINE FOR FINDING C*C(TRANSPOSE)=V BY SQUARE ROOT METHOD
C
      DIMENSION V(10,10),C(10,10),D(10,10),E(10,10)
      DO 20 I=1,M
      DO 20 J=1,M
      E(I,J)=0.0
      D(I,J)=0.0
   20 C(I,J)=0.0
      DO 30 I=1,M
   30 C(I,1)=V(I,1)/SQRT(V(1,1))
      DO 40 J=2,M
      DO 40 I=J,M
      IF(I-J)60,70,60
   70 L=I-1
      SS=0.
      DO 80 K=1,L
      SS=SS+C(I,K)**2


   80 CONTINUE
      C(I,J)=SQRT(ABS(V(I,J)-SS))


      GO TO 40
   60 SS=0.
      DO 90 K=1,L
   90 SS=SS+C(I,K)*C(J,K)
   50 C(I,J)=(V(I,J)-SS)/C(J,J)
   40 CONTINUE
      CALL GMTRA(C,D,10,10)
      CALL GMPRD(C,D,E,10,10,10)
      WRITE(3,140)
  140 FORMAT('1'20X,' THE SQUARE ROOT MATRIX C'///)
      DO 110 J=1,M
  110 WRITE(3,607)(C(I,J),I=1,M)
  607 FORMAT(/3(5X,10F10.5/))
      WRITE(3,150)
  150 FORMAT(////25X,' C TRANSPOSE C'/20X,' THIS SHOULD EQUAL THE VARIA
     1NCE COVARIANCE MATRIX'///)
      DO 100 J=1,M
  100 WRITE(3,607)(E(I,J),I=1,M)
      RETURN
      END
```

```
      SUBROUTINE GAUSS(/IX/,S,AM,/V/)
C
C     GENERATES RANDOM NORMAL NUMBERS
C
      COMMON SEED
      A=0.0
      DO 50 I=1,12
      Y=RANDOM(SEED)
   50 A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END
```

```
      SUBROUTINE ABVCO(NPQ,SDPQ,RINYR,R)
C
C     CALCULATES THE VAR-COVARIANCE MATRIX  R
C
      DIMENSION SDPQ(10),RINYR(10,10),R(10,10)
      DO 20 I=1,NPQ
      DO 20 J=1,NPQ
   20 R(I,J)=RINYR(I,J)*SDPQ(I)*SDPQ(J)
      RETURN
      END
```

```
      SUBROUTINE ABGPH(Y,YS,NO,STRT,ADD)
C     Y     = ACTUALS INPUT ARRAY
C     YS      ESTIMATES INPUT ARRAY
C     NO      LENGTH OF ABOVE ARRAYS, ALLOWABLE RANGE  8 TO 99
C     STRT    STARTING NUMBER ON X AXIS
C     ADD     SIZE OF INCREMENT FOR X AXIS
C             IE  STRT=20, ADD=1,  THEN X AXIS 20,21,22,23 ----
      DIMENSION OUT(19)
      DIMENSION Y(99),YS(99),LINE(99),NY(99),NYS(99)
C     INITIALISE LINE AS A BLANK SPACES AND NA, NB,ND AS CHARACTERS ANI
C     NC AS A BLANK
      DATA LINE,NA,NB,NC,ND/ 99*' ','*','.',' ','X'/
      DATA NY,NYS/99*0,99*0/
      T=0.
      B=32767
C     FIND MAXIMUM AND MINIMUM VALUES IN INPUT SUBROUTINE
      DO 80 M=1,NO
      IF(Y(M)-T)81,81,82
   82 T=Y(M)
   81 IF(YS(M)-T)83,83,84
   84 T=YS(M)
   83 IF(Y(M)-B)85,86,86
   85 B=Y(M)
   86 IF(YS(M)-B)87,80,80
   87 B=YS(M)
   80 CONTINUE
C     CALCULATE A-SCALING FACTOR SO Y AXIS FITS ON ONE PAGE OR 70 LINE
      SCALE=50/(T-B)
C     MULTIPLY Y AND YS BY SCALING FACTOR, ROUND OFF AND TRUNCATE TO
C     INTEGER NY,NYS
      DO 37 M=1,NO
      NY(M)=(Y(M)*SCALE)+.5
   37 NYS(M)=(YS(M)*SCALE)+.5
C     THUS CALCULATE MAX. AND MIN INTEGER VALUES WITH THE MIN VALUE NO
C     ROUNDED OFF.THEN CALCULATE THE ACTUAL NUMBER OF AXIS UNITS (NYAX
      KK=(T*SCALE)+.5
      KL=B*SCALE
      NYAXS=KK-KL+1
      WRITE(3,24)
   24 FORMAT('1 CUMULATIVE PROBABILITY DISTRIBUTION OF THE'//
     1' INTERNAL RATE OF RETURN'///)
C     CALCULATE NE TO MAKE MAXIMUM USE OF X AXIS
      NE=100/NO
C     FOR EACH TRUNCATED OBSERVATION SEE IF MAX. INTEGER THE SAME
C     IF IT IS PUT APPROPRIATE CHARACTER IN LINE
      DO 61 IA=1,NYAXS
      DO 40 MM=1,NO
      NVAL=NY(MM)
      NPAL=NYS(MM)
      IF(KK-NVAL)27,28,27
   28 LINE(MM)=NA
   27 IF(KK-NPAL)40,31,40
   31 IF(KK-NVAL)30,33,30
   30 LINE(MM)=NB
      GO TO 40
   33 LINE(MM)=ND
   40 CONTINUE
C     FIND THE DESCALING FACTOR FOR Y AXIS LABEL
      SK=KK
      SK=SK/SCALE
      JK=SK
C     WRITE THE LINE IN THE APPROPRIATE FORMAT DEPENDING ON NE
      GO TO(70,71,71,71,72,72,72,72,72,72,72,72,72,72),NE
   71 WRITE(3,56)SK,(LINE(NI),NI=1,NO)
   56 FORMAT(F10.2,' I',50(1X,A1))
      GO TO 60
   70 WRITE(3,58)SK,LINE
   58 FORMAT(F10.2,' I',100A1)
      GO TO 60
   72 WRITE(3,290)SK,(LINE(NW),NW=1,NO)
  290 FORMAT(F10.2,' I',20(4X,A1))
   60 CONTINUE
C     SET LINE BACK TO BLANKS
      DO 57 NI=1,99
      LINE(NI)=NC
   57 CONTINUE
```

```
C     REDUCE MAX INTEGER AND CONTINUE FOR NYAXS TIMES
      KK=KK-1
   61 CONTINUE
      WRITE(3,42)
   42 FORMAT(11X                 '-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
     1-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---',/)
      START=STRT
      DO 600 I=1,19
      OUT(I)=START
  600 START=START+ADD
      WRITE(3,520)OUT
  520 FORMAT(13X,19(F5.1)//)
      WRITE(3,521)
  521 FORMAT(50X,' INTERNAL RATE OF RETURN')
      WRITE(3,759)(Y    (I),I=1,NU)
  759 FORMAT(19F6.2//)
      RETURN
      END




      SUBROUTINE TRIAN(IX,PESS,AVER,OPT,OBS)
C
C     SIMULATES SAMPLES FROM A TRIANGULAR DISTRIBUTION
C
      CALL RANDU(IX,IY,RN)
      IX=IY
      AN=RN*(OPT-PESS)+PESS
      IF(AN-AVER) 2,2,4
    2 OBS=PESS+SQRT(RN*(OPT-PESS)*(AVER-PESS))
      GO TO 5
    4 OBS=OPT-SQRT((1-RN)*(OPT-PESS)*(OPT-AVER))
    5 RETURN
      END




      SUBROUTINE TRICR(IX,PESS,AVER,OPT,OBS,RN)
C
C     SIMULATES SAMPLES FROM A TRIANGULAR DISTRABUTION
C     WITH ALLOWANCE FOR CORRELATION
C
      IF(IX)10,10,20
   20 CALL RANDU(IX,IY,RN)
      IX=IY
   10 AN=RN*(OPT-PESS)+PESS
      IF(AN-AVER) 2,2,4
    2 OBS=PESS+SQRT(RN*(OPT-PESS)*(AVER-PESS))
      GO TO 5
    4 OBS=OPT-SQRT((1-RN)*(OPT-PESS)*(OPT-AVER))
    5 RETURN
      END
```